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Constructal optimization for an insulating wall combining heat flow, strength and volume ☆

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ABSTRACT

Volume is pursued smaller and smaller in engineering fields with various scales. Constructal optimization of a vertical insulating wall is carried out to meet three engineering requirements of heat insulation, mechanical strength and volume simultaneously in this paper. The optimization objective is the thermal resistance per unit volume, the global constraint is the fixed total mass (solid material) of the insulating wall, and the intersection of asymptotes method is employed. The results show that, for the specified number of air gaps, the maximal thermal resistance per unit volume decreases monotonically with the increase of the global Rayleigh number group, and the smaller the global Rayleigh number group, the quicker the decrease of the maximal thermal resistance per unit volume. For the specified global Rayleigh number group, the maximal thermal resistance per unit volume increases monotonically with the increase of the number of air gaps, but the increase of the maximal thermal resistance per unit volume is not obvious in small regions of the global Rayleigh number group. The optimal spacing of the air gap increases slowly with the increase of the number of air gaps. For the engineering case in which the volume of the insulating wall is limited strictly, this paper can provide some theoretical guidelines.

1. Introduction

What is the evolution mechanism of flows in nature and society? This problem is so interesting that it attracts many scholars in physics and life science strongly. In engineering science, designers pursue the strategy on how to arrange flows to obtain the best performance of the system. Constructal theory [1–8], which was put forward by Bejan [9] in 1996 and has become one of the most active fields in thermal science, is not only a deterministic physical principle for the evolution of natural flows (rivers, lungs, species, etc.) [1,3,10–15] and social dynamics [5,7], but also a scientific guideline for the optimization of engineering flows (heat, mass, fluid, etc.) [1–3,6,16–27]. The mentality core of constructal theory is constructal law [1,3,6,28] and it supplements the first and second laws of thermodynamics by expounding the flow configurations in non-equilibrium systems [1,3,6]. It is called constructal optimization [1,3,4,6] that employs constructal law to guide engineering design, i.e. to search the optimal internal structure, external shape and time rhythm of an object by taking performance maximization as an objective function with some given global constraints [29–40].

For every engineering optimization problem, the optimization objective is the necessary factor that should be specified for practical engineering requirements. One may obtain widely divergent results when different objective functions are employed. Single-objective optimization is to give prominence to engineering requirements; consequently, it may attend to one thing and lose another. Mauroy et al. [41] proved that the bronchial tree based on the minimization of fluid flow resistance is dangerous for animals. For giving attention to various engineering requirements, multi-objective and multidisciplinary optimizations emerge as the times require. In constructal optimization, thermal resistance and fluid flow resistance (pressure drop and pumping power) are usually taken into account simultaneously in multi-objective optimizations [1,6,14,42–45]. Multidisciplinary constructal optimization was pioneered by Lorente and Bejan [46], and the constructal optimization of an insulating wall by combining heat transfer and strength was studied in Ref. [46] where the constraint of fixed external dimensions was employed for the maximization of overall thermal resistance with specified mechanic strength, and the effects of the thickness of air gap and the number of air gaps on the overall thermal resistance were analyzed with the constraint of fixed

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Nomenclature

g	gravitational acceleration, m s^{-2}
H	height, m
I	area moment of inertia, m^4
k	thermal conductivity, W m^{-1} K^{-1}
L	spacing, m
m	exponent
n	number of air gap
Nu	Nusselt number
q	heat flow, W
R	overall thermal resistance, W K^{-1}
R^*	thermal resistance per unit volume, W K^{-1} m^{-3}
Ra	Rayleigh number
Ra^*	global Rayleigh number group, $Ra^* = LH^{-1}Ra_{H,\Delta T}^{1/4}$
V	volume, m^3
W	width, m

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	coefficient of volumetric thermal expansion, K^{-1}
ρ	density, kg m^{-3}
∇T	temperature difference, K
v	kinematic viscosity, m^2 s^{-1}

Superscript

~	dimensionless variables

Subscript

a	air
cond	conduction
conv	convection
max	maximum
opt	optimum
s	solid
0	reference

total mass (the amount of solid) of the insulating wall. After that, the multidisciplinary constructal optimizations were conducted for a beam under thermal attack [47], nano-scale fluid flow problem [48], an electromagnet with inserted high conductivity material [49,50], vascular structures [51–53], etc. For such problems, the key issues include multidisciplinary physical models, multi-objective optimization functions, global constraints and mathematical solutions. It is an interesting topic on how to combine thermal objectives and mechanical objectives with other engineering objectives in the pursuit of optimal flow configuration. Xie et al. [54] introduced a product function of heat flow and strength with power weight as a complex performance index of a vertical insulating wall, and performed the constructal optimization by taking the complex index maximization as the objective. Chen et al. [55] took three requirements of heat insulation, strength and weight into account simultaneously, and built the optimization objective function of thermal resistance per unit mass for constructal optimization of an insulating wall based on Ref. [46]. The work of Ref. [55] proposed an advisable scheme for the engineering cases in which the weight is limited strictly and met the need of pursuing the lighter weight. Alike, it should be noted that volume is also a very important factor in modern engineering and the smaller volume is pursued in many engineering fields with various scales, such as electronic chip, building, ship, space station, etc. Under the assumption of constant volume/mass of material used, Yu and Li [56] found that the fractal-like tree networks could significantly reduce the thermal conductivity, and a lower thermal conductivity matrix embedded with such a network could be a satisfactory insulating wall/composite. Based on Refs. [46,54,55], this paper will build an optimization objective function that is different from Ref. [55] and called as the thermal resistance per unit volume for meeting the three requirements of heat insulation, strength and volume simultaneously, and will investigate multidisciplinary constructal optimization for an insulating wall by utilizing the intersection of asymptotes method [57] that is also different from Refs. [46,55] in which the mechanical strength index was fixed at a certain constant for each numerical calculation. The work herein can provide an alternative design scheme of insulating walls, especially for the engineering cases in which the smaller volume is pursued.

2. Model

The model formulations presented herein are based on Ref. [46]. From the view of energy-saving technology, heat insulating [58,59] is equally as important as heat transfer enhancement. The insulating wall which serves as a classical low carbon technology has been used widely in civil buildings and various industries. An insulating wall with air cavities is one of them and it not only saves energy but also saves solid material. Fig. 1 shows the schematic diagram of a vertical insulating wall with air cavities [46]. The height H and the width W are the two external dimensions of the insulating wall. The solid material is slabbed into $n+1$ plies with equal thickness L_s equidistantly by n vertical air-filled gaps with equal spacing L_a. Herein, the total mass (the amount of solid) of the insulating wall is fixed as the global constraint for optimization. When the height H and the width W are all fixed, this constraint can be transferred so that the thickness of solid wall without air gap is fixed, i.e. [46]

$$L_0 = (n+1)L_s = Const. \tag{1}$$

The overall thickness of insulating wall with air gaps is [46]

$$L = nL_a + L_s(n+1). \tag{2}$$

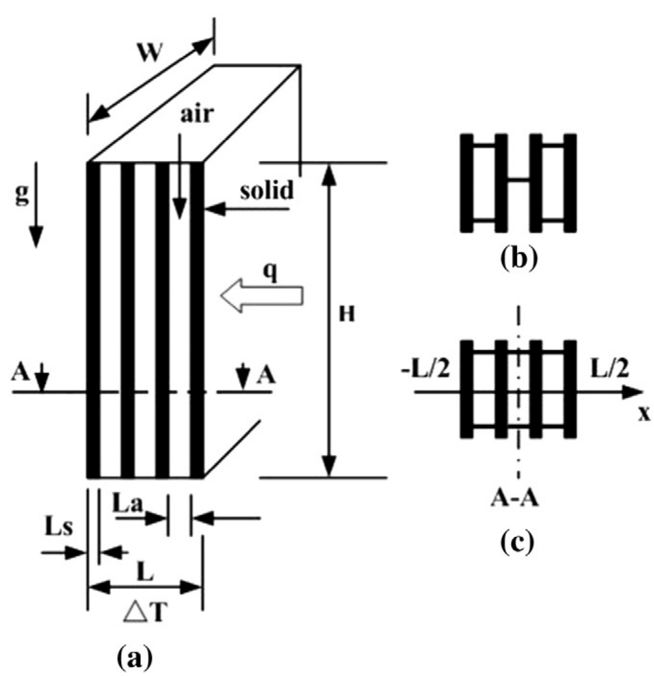

Fig. 1. Insulating wall [46].