International Communications in Heat and Mass Transfer 62 (2015) 33–36

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Imposition of the no-slip boundary condition via modified equilibrium distribution function in lattice Boltzmann method ☆

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ARTICLE INFO

Available online 7 February 2015

Keywords:
Immersed boundary technique
Bounce-back technique
Lattice Boltzmann method
Equilibrium boundary condition

ABSTRACT

A novel scheme for implementation of the no-slip boundary conditions in the lattice Boltzmann method is presented. In detail, we have substituted the classical bounce-back idea by the direct velocity boundary condition specification employing geometric-based manipulation of the equilibrium distribution functions. In this way we have constructed the equilibrium density function in such a way that it imposes the desired Dirichlet boundary conditions at numerical boundary points. Therefore, in fact a kind of equilibrium boundary condition is made. This specification for general curved solid surfaces is made by means of immersed boundary concepts, but without any need to interpolating density distribution values. On the other hand, the results show that the method presents a faster solution procedure in comparison to the bounce-back scheme.

1. Introduction

Classically, researchers have adopted the bounce-back scheme dealing with modeling the effects of solid boundaries in lattice-Boltzmann method (LBM) [1]. This scheme simply states reflection of density functions in the opposite direction, where facing a solid wall in the D_nQ_m configurations. One of the drawbacks of this technique is that it is at most first-order accurate unless the solid boundary is located exactly mid-way between the physical boundary point and the immediate node inside the solid [2]. This lack of accuracy propagates to the whole solution domain. In fact, the reflective rules of the density functions may implement the boundaries unphysically [2].

In order to remedy the shortcomings of the bounce-back scheme, several modifications have been recommended. Major modifications include attempts to increase the accuracy of the scheme to two [3] and amending the scheme for curved geometries via interpolation techniques [4–8]. Note that the idea of interpolation is limited in the sense that however the spatial accuracy is improved by higher-order interpolations, the time accuracy is essentially first-order [7].

In this study, inspired by the impressing idea of the immersed boundary methods (IBM) [8–12], we replace the traditional bounce-back scheme with the direct manipulation of the macroscopic velocities during the solution. This modification could yield more natural, simpler and faster implementation of the boundary conditions. In fact, the actual cost per boundary node in two-dimensions is only two velocity manipulation operations instead of nine reflective density corrections in e.g., a D_2Q_9 configuration. It also facilitates to explicitly imposing well-documented classical computational fluid dynamics boundary conditions (such as far field and free slip boundary conditions) in lattice-Boltzmann simulations.

2. Applying Dirichlet boundary condition using immersed boundary concept

In the cellular automata world, one devises simple physical laws and expects reasonable phenomena to emerge. LBM stems from cellular automata concept where the lattice Boltzmann equation (LBE) is the rule that states the evolution of density distribution function f of the particles at site x at time t moving with a velocity c_i during the time interval Δt along each lattice direction i. The LBE incorporating the single relaxation Bhatnagar–Gross–Krook (BGK) approximation has the form:

$$f_i(x + c\Delta t, t + \Delta t) - f_i(x,t) = \omega\left[f_i^{eq} - f_i(x,t)\right], \tag{1}$$

where ω denotes the relaxation factor and the local equilibrium distribution is an analog version of the Maxwellian distribution function for incompressible flow. In this context, the macroscopic flow properties are defined as $\rho = \sum f_i$ and $\rho u = \sum f_i c_i$.

The continuity equation and the Navier-Stokes equations can then be recovered with the second-order of accuracy from the Eq. (1) if the density variation is sufficiently small. In a typical simple LBM with D_2Q_9 configuration, Eq. (1) splits into two essential steps, namely collision and streaming phases. It is well-known that the collision process is fully local and the propagation of the distribution functions is uniform. In the most commonly used approach for simulating no-slip boundary

☆ Communicated by W.J. Minkowycz.

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2015.01.003
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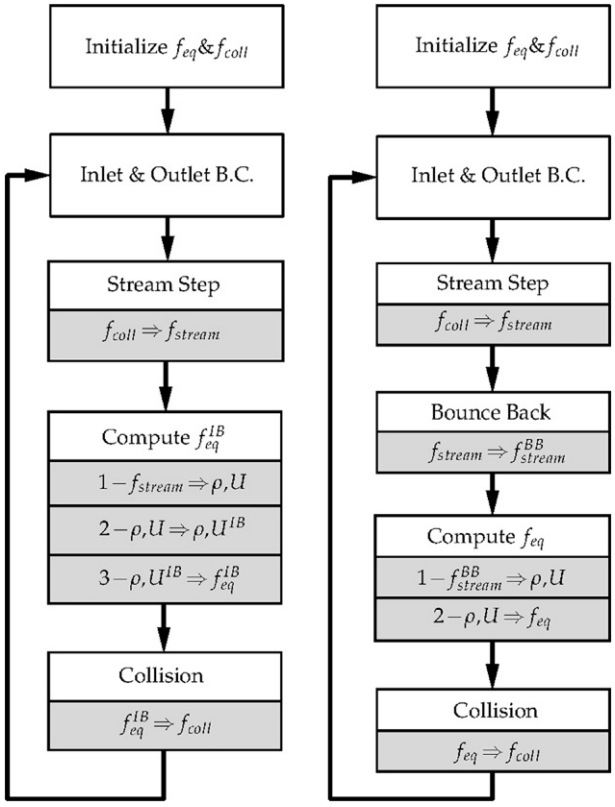

Fig. 1. Flowchart of the suggested method (left) and the classical bounce-back method (right).

condition known as bounce-back boundary condition, distribution functions are employed and their values for nodes adjacent to solid bodies are reflected in the desired directions.

In the presented method, we intend to manipulate the macroscopic velocity values for the implementation of the no-slip boundary condition. Before we show how the boundary condition is implemented, we note that one can adopt different strategies and configurations to construct a LBM algorithm loop. We have configured the algorithm in such a way that our suggested hierarchy (Fig. 1) enjoys a corresponding bounce-back counterpart, which is a standard implementation procedure.

Based on the flowchart, the process is as follows. Either in the classical bounce-back and our method, in the beginning of each step after common initialization (using equilibrium distribution function), we employ the equilibrium boundary conditions [3] for the plausible inlets and outlets. This step is applied for our first test case. Then, we let the distribution function f to stream. In the next step the bounce-back technique calculates the reflected values of f while instead, in the presented method we obtain the macroscopic values u and ρ from the streamed f by means of the following procedure to modify u to get u_{IB}. Now, we employ the boundary-conditioned velocity field u_{IB} to compute the boundary-conditioned equilibrium distribution function f_{eq}^{IB}. The key point is that f_{eq}^{IB} has sensed the effect of no-slip boundaries. The final stage is the application of the collision operator that uses f_{eq}^{IB} to get the updated value of f.

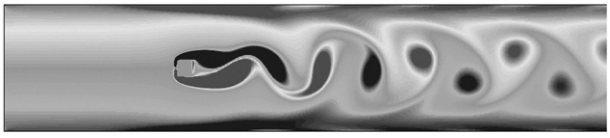

Fig. 2. Vorticity contours of the flow over a square for Re = 100.

For the two-dimensional general complex boundary case, we define the numerical boundary points as follows. A point N_B is a numerical boundary point associated with a line segment connecting two adjacent physical boundary points on the horizontal and vertical sides of the lattice if and only if

- it is inside the immersed solid region and
- at least one of its neighboring lattice nodes is a fluid node.

Now, we evaluate auxiliary moments at the numerical boundary points by a Taylor series expansion from the assumed exact values on the physical boundary points as

$$\rho(N_B) = \rho(P_B) + (l_n.\nabla)\sum_i f_i^{eq}, \tag{2}$$

$$\boldsymbol{j}(N_B) = \rho(N_B)\boldsymbol{u}(N_B) = \boldsymbol{j}(P_B) + (l_n.\nabla)\sum_i f_i^{eq}\boldsymbol{c}_i, \tag{3}$$

where, $\rho(P_B)$ and $\boldsymbol{j}(P_B)$ are the moments of the physical boundary point on the actual curve and l_n is the perpendicular distance from N_B to line connecting the horizontal and vertical intersections of the curved boundary with lattice sides.

These relations are generic for the one and two-dimensional problems. Note that, apparently these intersection points are not resolved by the Cartesian grids and therefore $\rho(P_B)$ and $\boldsymbol{j}(P_B)$ can only be employed to approximate the equilibrium distribution functions in our target numerical boundary points. We apply this scheme to force the flow system to encounter the fraction which solid body has occupied in the cut-cell. The idea of expanding the physical boundary values to the stair-step shapes has been previously applied with success for improving the mass conservations of a Navier–Stokes flow interacting with curved boundaries [13].

Now, we derive the fictitious equilibrium distribution function on the numerical boundary points by means of maximizing the relative entropy densities. The necessary condition for the total functional $\widetilde{S}$ i.e., directional functional representing the relative entropy densities to have extremum is

$$\frac{\partial \hat{S}}{\partial f_i^{eq}} = \frac{\partial \widetilde{S}_x}{\partial f_i^{eq}} + \frac{\partial \widetilde{S}_y}{\partial f_i^{eq}} = -2C\left[\ln\frac{f_i^{eq}}{W_i} + 1\right] + \left[\widetilde{A}_x\frac{l_x}{d_x}(-1) + \widetilde{A}_y\frac{l_y}{d_y}(-1)\right] + \boldsymbol{c}_i.\left[\widetilde{\boldsymbol{B}}_x\frac{l_x}{d_x}(-1) + \widetilde{\boldsymbol{B}}_y\frac{l_y}{d_y}(-1)\right] = 0 \tag{4}$$

where, C is a molecular constant, l_x and l_y are the Cartesian components of l_n, $\tilde{A}$ and $\widetilde{\boldsymbol{B}}$ are weights to be determined and d_x and d_y are the horizontal and vertical distances from N_B to cut-cell locations in horizontal and vertical directions respectively. Therefore, taking $A = \frac{C}{2}\left\{-\widetilde{A}_x\frac{l_x}{d_x} - \widetilde{A}_y\frac{l_y}{d_y}\right\} - 1$, $\boldsymbol{B} = \frac{C}{2}\left\{-\widetilde{\boldsymbol{B}}_x\frac{l_x}{d_x} - \widetilde{\boldsymbol{B}}_y\frac{l_y}{d_y}\right\} - 1$, we can find the general solution by the Taylor series expansion around $\boldsymbol{j} = 0$ as

$$f_{eq}^{IB} = W_i e^{A_0}\left\{1 + B_1 c_i.\boldsymbol{j} + \frac{B_1^2}{2}(\boldsymbol{c}_i.\boldsymbol{j})^2 + A_2\boldsymbol{j}^2\right\}. \tag{5}$$

This modified equilibrium distribution function is employed in our algorithm for numerical boundary points.

3. Results

In order to examine the method, we first bring the results for the classical flow over a squared cylinder in a confined channel with a parabolic inlet velocity profile. Here, the geometric configuration is identical to that of [3]. Then, we have compared the findings with the