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A fractal model for spherical seepage in porous media ☆

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ABSTRACT

The characters of spherical/radial seepage in porous media have attracted steadily attention in many sciences and technologies such as in oil/water exploration, nuclear waste disposal, and heat and mass transfer in aerospace materials, biological tissue and organs. Considering the effects of capillary pressure, in this work we study the effective radial permeability and relative permeability for spherical seepage in porous media by applying the fractal theory and technique for porous media. The proposed models are related to the structural parameters of porous media, such as fractal dimensions, porosity, tortuosity and fluid properties. The validity of the proposed model is verified by comparing the model predictions with the available model and the existing experimental data. The present results show that the effective radial permeability and radial porosity decrease with the increase of radial distance. The parametrical effects on the radial permeability are also studied. It is found that the relative permeability for spherical seepage of wetting phase increases with the increase of wetting phase saturation, and the relative permeability of the non-wetting phase decreases with the increase of the wetting phase saturation. The contributions for permeability and relative permeability for spherical/radial seepage from capillary pressure can be negligible when $p_{c,av}/p_m < 0.01$, otherwise, the effect of capillary pressure on the seepage in porous media should be taken into account.

1. Introduction

Knowledge of the characters of spherical/radial seepage in porous media plays an important role in oil/water exploration, nuclear waste disposal, and heat and mass transfer in aerospace materials, biological tissue and organs [1–5]. Early as in 1940s, Hurst [6] and Van Everdingen and Hurst [7] proposed the mathematical processing method for spherical seepage, leading to the foundation in oil exploration. Chatas [8] studied the unstable-state spherical seepage and got the exact solutions under the infinite, limited and a constant pressure by using three kinds of boundaries with Laplace transform. Brigham et al. [2] analyzed the spherical seepage in drilling well with the same way. Furthermore, Jeffrey and Koederitz [9] studied the spherical seepage problems by considering the skin effect. However, their results were not involved in the microstructures of porous media, such as pore sizes and their distribution.

Due to the disordered and complicated micro-structures of porous media, it is very difficult to analytically study the seepage characteristics in porous media. Fortunately, the micro-structures of porous media have been shown to have the self-similarity and fractal characteristics [10,11], and the fractal geometry theory has received the great success in analysis of the flow and transport properties in porous media. Many researchers studied the permeability, imbibition and starting pressure gradient in porous media by using the fractal geometry theory and technique. Yu and Cheng [12] proposed a fractal model to investigate the seepage characteristics in saturated porous media and obtained the analytical expressions for permeabilities of particle-like porous media and porous fabrics by assuming that porous media consist of a bundle of tortuous capillaries, whose size distribution follows the fractal scaling law. Later, Liu and Yu [13,14] got the permeability and relative permeability of porous media with capillary pressure included. Recently, Wang and Yu [15] studied the relative permeability of unsaturated porous media embedded with a fractal-like tree branched networks by considering the capillary pressure. Xiao et al. [16] studied the relative permeability of unsaturated fractal porous media with the Monte Carlo method. Xu et al. [17] investigated the relative permeability in unsaturated porous media by assuming that all capillaries with the radii less than a critical radius are saturated, and the others with the radii larger than the critical radius are unsaturated, then fractal-Monte Carlo simulations were performed for the relative permeability. Li and Horne [18] investigated the drainage and imbibition in porous media by using the fractal geometry and the Brooks–Corey capillary pressure model. Cai et al. [19,20] used the fractal geometry method to derive analytical

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model for spontaneous imbibition in porous media. Recently, Cai et al. [21] developed a generalized fractal model for spontaneous imbibition in porous media with shapes of pores included. Yun et al. [22] studied Bingham fluid flow characteristics such as starting pressure gradient and permeability in porous media. Porous membranes have been used in many applications, such as ultra-filtration, pervaporation, distillation and gas diffusion layer (GDL) in proton exchange membrane fuel cells (PEMFCs). Zhang [23], Othman and Helwani [24], R. Wu et al. [25], and B.Q. Xiao et al. [26] applied the fractal theory for porous media and the parallel capillary model to analyze the permeabilities of water flow and gas diffusion in porous membranes. Hao and Cheng [27] used the Lattice Boltzmann simulations and the fractal theory for porous media as well as the parallel capillary model to simulate the anisotropic permeabilities in carbon paper gas diffusion layers.

In addition to the above brief review, investigations on radial seepage in porous media have also attracted much attention in the past decades. Barker [28] developed a generalized radial flow (GRF) model for porous media with standard test conditions: constant rate, and this model successfully reflects the information about geometry of aquifer from pumping test data. However, the parameter in the GRF model: the flow dimension was unclear in physical meaning and difficult to be obtained. Barker thought that it may related to flow in fractal porous media. Chang and Yortsos [29] proposed a model for the radial seepage in saturated porous media by applying the fractal geometry theory and technique. However, their model was not rigorously derived, and it was obtained by analogy of diffusion on the Sierpinski gasket with the random fractal porous medium. Later, Acuna and Yortsos [30] used the model to study the pressure change around the well in natural reservoir. Leveinen [31] assumed that the grounder water flow is laminar in fractured rock and developed a typical curve method to estimate the hydraulic properties of the radial flow from pumping tests in saturated fractured media. Song and Liu [32] obtained an approximate analytical solution for power law fluid for spherical seepage in saturated porous media and studied the starting pressure gradient and dynamic pressure. However, the model was not related to the microstructures of porous media. Walker et al. [33] carried out the pumping tests on fractal porous media and examined flow dimension in GRF model with three stochastic models including Monte Carlo analysis, and they found that flow dimension may be useful for the parameters to be selected. However, the calculation is time-consuming. Yun et al. [34] derived the analytical expression for radial seepage permeability by fractal theory for saturated porous media with constant porosity along radial direction. Sheng et al. [35] obtained a model for the unsteady-state spherical seepage with fractal geometry methods. However, the tortuosity of flow path was not considered in their model. Xu et al. [36] used the fractal theory and the Monte Carlo method to study the plane-radial seepage in saturated porous media.

In order to well understand the mechanisms of spherical (three-dimensional) seepage in unsaturated porous media, in this work an analytical model is presented based on the fractal geometry theory for porous media with capillary pressure included and with annular flow in each capillary. The effect of capillary pressure on the radial permeability is studied in detail. In Section 2, we will introduce the fractal theory for porous media. In Section 3, the analytical expressions are derived regarding the radial effective permeability with the effect of capillary pressure for the spherical seepage based on the fractal geometry theory for unsaturated porous media. In addition, the relative permeabilities of wetting and non-wetting phases for spherical seepage with capillary pressure are obtained in Section 4. The validity of the proposed model is verified by the comparisons between the present model predictions and those from the available model as well as the existing experimental data, and the effects of parameters such as radial distance, fractal dimensions and diameters of particles on the spherical seepage characters in porous media are discussed in Section 5. The conclusions from this work are given in Section 6.

2. Fractal theory for spherical seepage in porous media

Spherical seepage in three dimensions widely exits in oil/water/gas reservoirs, where the flow paths are converged in three dimensions towards a common center (see Fig. 1).

Based on the fractal geometry theory for porous media, in a representative unit cell on the interface of a spherical porous medium, the cumulative number of capillaries whose pores sizes are greater than or equal to λ is given by the following fractal scaling law [12,37]

$$N(L \geq \lambda) = (\lambda_{max}/\lambda)^{D_f} \tag{1}$$

where N is the number of pores or capillaries, λ_{max} is the maximum capillary diameter and D_f is the fractal dimension for pore space, and in general, $0 < D_f < 2$ in two dimensions and $0 < D_f < 3$ in three dimensions.

Differentiating Eq. (1) with respect to λ results in the number of capillaries/pores whose diameters are in the interval of $d\lambda$ near λ, i.e.

$$-dN = D_f \lambda_{max}^{D_f} \lambda^{-(D_f+1)} d\lambda \tag{2}$$

where $-dN > 0$, which indicates that the number of capillaries/pores decreases with the increase of capillary/pore sizes.

It has been shown that in fractal porous media, the following relation is satisfied [37]

$$(\lambda_{min}/\lambda_{max})^{D_f} \cong 0. \tag{3}$$

Generally, in porous media $\lambda_{min}/\lambda_{max} \sim 10^{-2}$.

Based on Eq. (2), the total pore area is obtained by

$$A_p = -\int_{\lambda_{min}}^{\lambda_{max}} \frac{\pi\lambda^2}{4} dN = \frac{\pi D_f \lambda_{max}^2 (1-\phi)_0}{4\left(2-D_f\right)} \tag{4}$$

where ϕ_0 is the area porosity, and D_f is the fractal dimension for pore space.

The total cross-sectional area of the representative unite cell for the pores/capillaries is calculated by

$$A_0 = \frac{A_p}{\phi_0} = \frac{\pi D_f \lambda_{max}^2}{4\left(2-D_f\right)} \frac{1-\phi_0}{\phi_0}. \tag{5}$$

The porosity and the fractal dimension in representative unit cell on the interface of the spherical porous medium are related by [12,37]

$$\phi_0 = (\lambda_{min}/\lambda_{max})^{d_E - D_f} \tag{6}$$

where d_E is the Euclid dimension, and $d_E = 2$ in two dimensions.

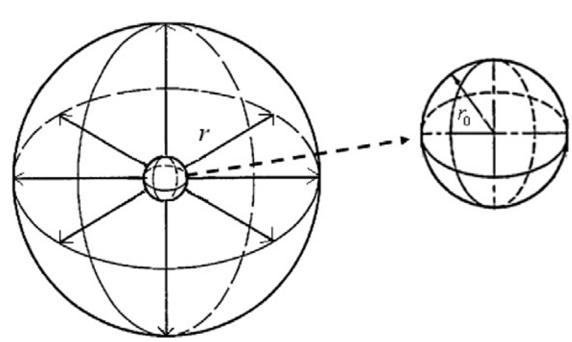

Fig. 1. Schematic of spherical seepage in three dimensions [9], where r is the radius for sphere seepage and r_0 is the radius for internal sphere filled with fluid.