International Communications in Heat and Mass Transfer 58 (2014) 193–199

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Heat transfer analysis of the peristaltic instinct of biviscosity fluid with the impact of thermal and velocity slips☆

Noreen Sher Akbar [a,*], Z.H. Khan [b]

[a] DBS&H CEME, National University of Sciences and Technology, Islamabad, Pakistan
[b] Department of Mathematics, University of Malakand, Chakdara, Dir (Lower), Khyber Pakhtunkhwa, Pakistan

ARTICLE INFO

Available online 16 September 2014

Keywords:
Inclined magnetic field
Thermal slip
Velocity slip
Peristaltic flow
Irregular channel
Biviscosity fluid

ABSTRACT

In this article, we have discussed inclined magnetic field on the peristaltic flow of biviscosity with the impact of thermal and velocity slips in an irregular channel. The momentum governing equations are fabricated under long wavelength and low Reynolds number guesstimate. Exact solutions are found in closed-form for flow equations representing stream function and pressure gradient. The flow comportment is examined over the properties of the governing parameters such as, the upper limit apparent viscosity coefficient β, slip parameters Λ, γ, Hartmann number M and amplitudes ϕ, a, b and d on pressure rise, velocity, pressure gradient and subsequently on streamlines.

1. Introduction

Peristaltic transport is a subject of scientific and engineering research during the past few decades. Peristalsis is a kind of fluid transport induced by a progressive wave of area contraction or expansion along the walls of a distensible duct containing liquid. In physiology, it has been found to be involved in many biological organs such as ureter, gastrointestinal tract, duct's afferents of the male reproductive tracts, cervical canal, female fallopian tube, lymphatic vessels and small blood vessels. In addition, peristaltic pumping occurs in many practical applications involving biomechanical systems such as roller and finger pumps. Several theoretical and experimental studies [1–5] have been conducted to understand peristaltic flow. The literature on peristalsis is quite extensive.

A mathematical model based on viscoelastic fluid (fractional Oldroyd-B model) flow for the peristaltic flow of chyme in the small intestine is studied by Tripathi [6]. According to him the size of trapped bolus reduces with increasing the amplitude ratio whereas it is unaltered with other parameters. Ebaid [7] presents numerical treatment for the solution of the hydromagnetic peristaltic flow of a bio-fluid with variable viscosity in a circular cylindrical tube using the Adomian decomposition method. He present closed form solution for velocity profile. Very recently Akbar et al. [8] discussed peristaltic flow of a Carreau nanofluid in an asymmetric channel with numerical simulation. They transformed governing nonlinear partial differential equations into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the fourth and fifth order Runge–Kutta–Fehlberg. Very few articles in peristaltic literature discussed slip effects. It has been experimentally verified that the non-Newtonian fluids such as polymer, polymer melts often exhibits macroscopic wall slips which in general can be described by nonlinear and non-monotone relation between the slip velocity and traction see Refs. [9–15]. Further recent research could be viewed in Refs. [16–20].

Considering the above importance of peristaltic flow of non-Newtonian fluids with thermal and velocity slips, we have presented the peristaltic flow of a biviscosity fluid in an irregular channel with velocity and thermal slips. To the best of the author's knowledge no attempt has been made in peristaltic literature for biviscosity fluid in an irregular channel. The governing equations for proposed fluid model have been modeled in Cartesian coordinated. Exact solutions have been evaluated for velocity, pressure gradient and temperature in closed-form. The behavior of velocity, temperature, pressure gradient, pressure rise and streamlines for different physical parameters is presented graphically.

2. Flow equations

We consider an incompressible biviscosity fluid in an irregular channel under influence of inclined MHD with channel width $d_1 + d_2$. Velocity and thermal slip conditions are also taken into account. Sinusoidal wave propagating with constant speed c along the walls of the channel. The somatic model and harmonize system are shown in Fig. 1.

☆ Communicated by W.J. Minkowycz.

* Corresponding author.
E-mail address: noreensher1@gmail.com (N.S. Akbar).

http://dx.doi.org/10.1016/j.icheatmasstransfer.2014.08.036
0735-1933/

Nomenclature

U,V	Velocity components
M	Hartmann number
X,Y	Coordinates
a_1, b_1	Wave's amplitudes
β	Apparent viscosity coefficient
$\mu\beta$	Plastic viscosity
μ	Viscosity of the fluid
γ	Thermal slip parameter
ν	Kinematic viscosity of the fluid
t	Time
c	Wave speed
ϕ	Phase difference
F	Flow rate
p_y	Yield stress
Λ	Velocity slip parameter

Irregularity in the channel flow is due to the following wall surface expressions:

$$Y = \overline{H}_1 = d_1 + a_1 \cos\left[\frac{2\pi}{\lambda}(X - ct)\right],$$
$$Y = \overline{H}_2 = -d_2 - b_1 \cos\left[\frac{2\pi}{\lambda}(X - ct) + \phi\right]. \tag{1}$$

The expression for fixed and wave frames is related by the following relations

$$\bar{x} = \overline{X} - ct, \quad \bar{y} = \overline{Y}, \quad \bar{u} = \overline{U} - c, \quad \bar{v} = \overline{V}, \quad p(\bar{x}) = P(\overline{X}, t). \tag{2}$$

The constitutive equation for incompressible biviscosity fluids [5] is defined as follows

$$\bar{S}_{ij} = \begin{cases} 2\left(\mu_\beta + p_y/\sqrt{2\pi}\right)e_{ij}, & \pi > \pi_c, \\ 2\left(\mu_\beta + p_y/\sqrt{2\pi_c}\right)e_{ij}, & \pi < \pi_c, \end{cases} \tag{3}$$

$\pi = e_{ij}$; e_{ij} is the (i,j)component of deformation rate, which is

$$e_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial yi}\right). \tag{4}$$

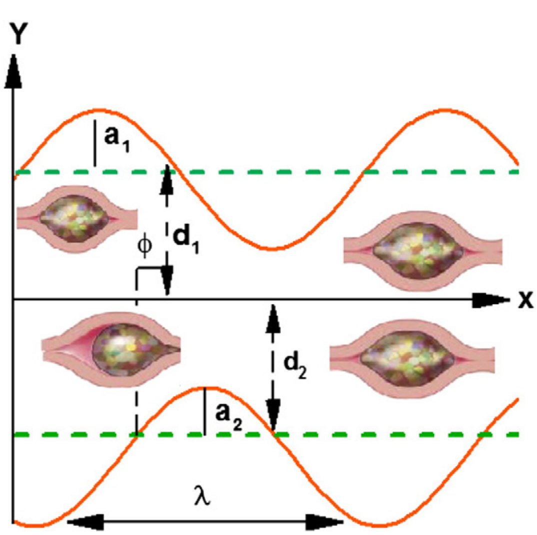

Fig. 1. Diagram of the projected model geometry.

Introduce the following non-dimensional quantities

$$x = \frac{2\pi\bar{x}}{\lambda},\ y = \frac{\bar{y}}{d_1},\ u = \frac{\bar{u}}{c},\ v = \frac{\bar{v}}{c},\ t = \frac{2\pi\bar{t}}{\lambda},\ \delta = \frac{2\pi d_1}{\lambda},\ d = \frac{d_2}{d_1},\ P = \frac{2\pi d_1^2 P}{\mu c\lambda},$$
$$h_1 = \frac{\bar{h}_1}{d_1},\ h_2 = \frac{\bar{h}_2}{d_2},\ Re = \frac{\rho c d_1}{\mu},\ a = \frac{a_1}{d_1},\ b = \frac{a_2}{d_1},\ d = \frac{d_2}{d_1},\ S = \frac{\bar{S} d_1}{\mu c},\ \beta = \mu_\beta\sqrt{2\pi_c}/p_y. \tag{5}$$

The stream function and velocity field are related by the expressions

$$u = \frac{\partial \Psi}{\partial y},\ v = -\delta\frac{\partial \Psi}{\partial x}. \tag{6}$$

Under the long wavelength and low Reynolds number assumption, the dimensionless governing equations take the following form

$$\left(\left(1 + \frac{1}{\beta}\right)\frac{\partial^4 \psi}{\partial y^4} - M^2 Cos^2\Theta\frac{\partial^2 \psi}{\partial^2 y}\right) = 0, \tag{7}$$

$$\frac{dP}{dx} = \frac{\partial}{\partial y}\left[\left(\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 \psi}{\partial y^2} - M^2 Cos^2\Theta(\psi + 1)\right)\right], \tag{8}$$

$$\frac{\partial^2 \theta}{\partial y^2} = -B_r\left[\frac{1}{\beta}\left(\frac{\partial^2 \Psi}{\partial y^2}\right)^2\right], \tag{9}$$

subject to the boundary conditions

$$\Psi = \frac{F}{2},\ \frac{\partial \Psi}{\partial y} = -\Lambda\left(\frac{1}{\beta}\frac{\partial^2 \psi}{\partial y^2}\right) - 1,\ \text{at } y = h_1 = 1 + a\cos x,$$
$$\Psi = -\frac{F}{2},\ \frac{\partial \Psi}{\partial y} = \Lambda\left(\frac{1}{\beta}\frac{\partial^2 \psi}{\partial y^2}\right) - 1,\ \text{at } y = h_2 = -d - b\cos(x + \varphi), \tag{10}$$

$$\theta + \gamma\frac{\partial \theta}{\partial y} = 0, \quad \text{at } y = h_1 \quad \theta - \gamma\frac{\partial \theta}{\partial y} = 1, \quad \text{at } y = h_2. \tag{11}$$

The flow rates in fixed and wave frames are related by [14]

$$Q = F + 1 + d. \tag{12}$$

3. Exact solutions

The moment Eq. (7) for the proposed model is a fourth order linear differential equation, which has a closed-form solution

$$\Psi(x,y) = C_1 + C_2 y + C_3 e^{\frac{M\cos\Theta\sqrt{\beta}y}{\sqrt{\beta+1}}} + C_4 e^{-\frac{M\cos\Theta\sqrt{\beta}y}{\sqrt{\beta+1}}}. \tag{13}$$

The constants of integration C_i, $i = 1, 2, 3$, and 4 are obtained using boundary conditions (9) through Mathematica 9. A closed form expression for the pressure gradient is obtained by substituting Eq. (13) into Eq. (8),

$$\frac{dP}{dx} = -M^2\cos\Theta C_2, \tag{14}$$

where $C_2(\beta, \phi, M, a, b, d, x, Q, \gamma, \Lambda)$. The dimensionless pressure rise ΔP is obtained by substituting Eq. (11) into the following equation

$$\Delta P = \int_0^1 \left(\frac{dP}{dx}\right)dx. \tag{15}$$