International Communications in Heat and Mass Transfer 52 (2014) 26–32

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Constructal entransy dissipation rate minimization for variable cross-section insulation layer of the steel rolling reheating furnace wall☆

Huijun Feng, Lingen Chen *, Zhihui Xie, Fengrui Sun

Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
Military Key Laboratory for Naval Ship Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China

ARTICLE INFO

Available online 14 January 2014

Keywords:
Constructal theory
Entransy dissipation rate
Variable cross-section insulation layer
Thermal insulation process
Steel rolling reheating furnace
Generalized thermodynamic optimization

ABSTRACT

A variable cross-section (distributed thickness and width) insulation layer of the steel rolling reheating furnace wall is investigated subjected to the constraints of the total volume and cross-sectional area of the insulation material. According to the entransy dissipation extremum principle of the thermal insulation process, the thickness of the insulation layer is optimized by taking minimum entransy dissipation rate as optimization objective, and the optimal construct of the insulation layer is obtained. The results show that when the temperature distribution of the furnace is linear with the length, the optimal thickness of the insulation layer with minimum entransy dissipation rate is linear with the dimensionless longitudinal position, which is evidently different from that with minimum heat loss rate. When the dimensionless temperature at the low temperature side $\varepsilon = 0$, the minimum entransy dissipation rate of the insulation layer with distributed thickness is decreased by 33.33% than that with uniform thickness, and is decreased by 8.85% than that based on minimum heat loss rate. Essentially, the temperature gradient field obtained based on minimum entransy dissipation rate is more homogenous than that based on minimum heat loss rate, and the corresponding thermal stress performance is better. The decrement of the entransy dissipation rate tends to increase for the exponential temperature distribution case with a large exponent. Moreover, the insulation layer with triangular cross-section has a better global thermal insulation performance derived from entransy dissipation than those with rectangular and trapezoidal cross-sections. Therefore, the optimal construct obtained by adopting variable cross-section insulation layer (distributed thickness and width) and based on minimum entransy dissipation rate can improve the global thermal insulation performance of the insulation layer derived from entransy dissipation, and can reduce its average heat loss rate defined based on entransy dissipation simultaneously. The optimal construct obtained based on minimum entransy dissipation rate can provide a new scheme for the design of practical thermal insulation system different from that based on minimum heat loss rate, which can satisfy the different requirements in the design of practical thermal insulation systems.

1. Introduction

Heat transfer enhancement problem [1] is a hotspot in heat transfer field, and thermal insulation problem is another important problem needed to pay attention to. Many scholars have shown great interest in thermal insulation problems [2–11]. Bejan [2] optimized the insulation layer thicknesses of the steel rolling reheating furnaces based on minimum heat loss rate, and the results showed that the thermal insulation performances of the insulation layers could be greatly improved. Kang et al. [3] further investigated the optimal insulation layer distributions of the steel rolling reheating furnaces with multi-layer and variable cross-section ones, and the results showed that great reduction of the heat loss rate of the insulation layer could be derived when the furnace wall temperature distribution tended to be more convex. Feng et al. [4] proposed the entransy dissipation extremum principle for thermal insulation process, and carried out constructal optimizations of a single plane and cylindrical insulation layers as well as multi-layer insulation layers of the steel rolling reheating furnace walls based on this principle. The results showed that the optimal constructs of the insulation layers obtained were different from those based on minimum heat loss rate. Kalyon and Sahin [5–7] investigated the optimal insulation distributions of the fluid flow pipes, and the results showed that different insulation boundary conditions had great effects on the optimal distributions of the insulation layers.

☆ Communicated by P. Cheng and W.Q. Tao.

* Corresponding author at: College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China.
E-mail addresses: lgchenna@yahoo.com, lingenchen@hotmail.com (L. Chen).

0735-1933/$ – see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.icheatmasstransfer.2014.01.006

Nomenclature

A	Cross-sectional area of the insulation material, m^2
b	Taper of the trapezoidal cross-section
$\dot{E}_{h\phi}$	Entransy dissipation rate per unit volume, $W \cdot K/m^3$
E_{vh}	Entransy, $J \cdot K$
$\dot{E}_{vh\phi}$	Entransy dissipation rate, $W \cdot K$
k	Thermal conductivity, $W/K/m$
L	Length of the insulation layer, m
n	Exponent
p	Width, m
Q_{vh}	Thermal capacity, J
q	Heat loss rate, W
T_0	External temperature of the insulation layer, K
t	Thickness, m
V	Volume of the insulation material, m^3
v	Control volume

Greek symbols

∇T	Temperature gradient, K/m
λ	Lagrange multiplier
ξ	Dimensionless longitudinal position

Subscripts

avg	Average
c	Uniform thickness
m	Minimal
opt	Optimal
p	Uniform width
ε	Dimensionless temperature

Lorente and Bejan [8] carried out constructal optimization of a vertical insulating wall by combining flow and strength, and pointed out that the internal structure of the wall could be optimized by considering heat transfer and mechanical strength simultaneously. Xie et al. [9] and Chen et al. [10] further optimized the internal structures of the walls discussed in Ref. [8] by combining heat flow, strength and weight simultaneously. Du et al. [11] considered the optimal porosity distribution of fibrous insulation, and the results showed that the continuous optimal porosity distribution corresponded to a high porosity at each boundary and a low porosity in the center.

To describe the heat transfer ability of an object, Guo et al. [12,13] defined a new physical quantity, entransy, and then put forward the extremum principle of entransy dissipation for heat conduction process. Henceforth, many scholars have done various works about heat transfer optimizations based on entransy theory [14–28], which enriches the optimization theory of heat transfer field.

Constructal law [29,30] is a powerful theory in illustrating and solving various design problems [31–52]. In the later work, constructal theory was successfully applied into the optimal distributions of the heaters [51,52] and insulation layers [3,4] of the reheating furnaces. Based on constructal theory and the insulation layer model in Ref. [3], the variable cross-section insulation layer model of the steel rolling reheating furnace will be studied by taking minimum entransy dissipation rate as optimization objective in this paper. The optimal distribution of the insulation layer will be obtained based on entransy dissipation extremum principle for thermal insulation process, and the performance comparison between the optimal constructs of the insulation layer based on the minimizations of entransy dissipation rate and heat loss rate will be carried out.

2. Definition of entransy dissipation rate [12]

Entransy, which is a new physical quantity reflecting heat transfer ability of an object, was defined in Ref. [12] as

$$E_{vh} = \frac{1}{2} Q_{vh} U_h = \frac{1}{2} Q_{vh} T \tag{1}$$

where $Q_{vh} = Mc_v T$ is thermal capacity of an object with constant volume, U_h or T represents the thermal potential. The entransy dissipation function, which represents the entransy dissipation per unit time and per unit volume, was deduced as [12]

$$\dot{E}_{h\phi} = -\dot{q} \cdot \nabla T = k(\nabla T)^2 \tag{2}$$

where $\dot{q}$ is thermal current density vector, and ∇T is the temperature gradient.

The entransy dissipation rate of the whole volume is

$$\dot{E}_{vh\phi} = \int_v \dot{E}_{h\phi} dv = \int_v |\dot{q} \cdot \nabla T| dv = \int_v k(\nabla T)^2 dv \tag{3}$$

where v is the control volume.

According to the entransy concept mentioned above and analogizing with the extremum principle of entransy dissipation for heat conduction process [12], Feng et al. proposed the entransy dissipation extremum principle for thermal insulation process, which was stated as [4]: for a fixed boundary heat flux (heat loss) with certain constraints, the thermal insulation process is optimized when the entransy dissipation is maximized (maximum average temperature difference), while for a fixed boundary temperature, the thermal insulation process is optimized when the entransy dissipation is minimized (minimum average heat loss rate defined based on entransy dissipation, which is defined as the entransy dissipation rate divided by temperature difference). For the fixed boundary temperature and based on entransy dissipation extremum principle for thermal insulation process, the constructal optimization of the variable cross-section thermal insulation system will be carried out with minimum entransy dissipation rate in this paper. When the entransy dissipation rate of the insulation layer reaches its minimum, the average heat loss rate defined based on entransy dissipation will reach its minimum at the same time. The temperature gradient field and heat flux density (for a constant thermal conductivity) of the insulation layer tend to be homogenous [4,13], the thermal performance in the global field of the insulation layer is considered, and the corresponding thermal stress performance is better. When the heat loss rate of the insulation layer reaches its minimum, the total heat loss is reduced. However, the temperature gradient field and heat flux density of the insulation layer are not homogenous, and the temperature gradient and heat flux in a part field of the insulation layer may be large. Therefore, the thermal performance in the each field of the insulation layer is not completely considered, and the corresponding thermal stress performance is not perfect compared with the former one. From this point of view, the global thermal insulation performance of the insulation layer derived from entransy dissipation is better.

3. Constructal optimization of variable cross-section insulation layer

A one-dimensional insulation layer model of a furnace wall is shown in Fig. 1 [3], and its typical application is the steel rolling reheating furnace in the iron and steel industry. The billet steel is heated by the hot gas in the inner of the hearth, and the heat is emitted through the insulation layer of the furnace wall. For simplification, the temperature distribution of the internal furnace wall along the x-direction (from the cold end $x = 0$ to the hot end $x = L$) is assumed to be known, i.e., $T(x)$