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An inverse hyperbolic heat conduction problem in estimating base heat flux of two-dimensional cylindrical pin fins ☆

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ABSTRACT

In this study, an inverse algorithm based on the conjugate gradient method and the discrepancy principle is applied to solve the inverse hyperbolic heat conduction problem in estimating the unknown space- and time-dependent base heat flux of a cylindrical pin fin from the knowledge of temperature measurements taken within the medium. The inverse solutions have been justified based on the numerical experiments in which three specific cases to determine the unknown base heat flux are examined. The temperature data obtained from the direct problem are used to simulate the temperature measurements. The influence of measurement errors upon the precision of the estimated results is also investigated. Results show that an excellent estimation on the space- and time-dependent base heat flux can be obtained for the test cases considered in this study.

1. Introduction

Fourier's law has been traditionally the mainstream theory used to solve heat conduction problems. Although Fourier's law bears a theoretical flaw that thermal signal travels at an infinite speed, it solves most large time and/or length scales engineering heat conduction problems with satisfactory accuracy. Yet the development of laser heating and nanotechnology has created heat conduction problems within very small time and/or length scales. For example, a carefully controlled incident beam can be used to heat up a very small patch of area at a rate up to 180 K/s for a few nanoseconds [1]. In such situations, researchers have reported that the predictions by Fourier heat conduction do not agree well with experimental observations. Maurer and Thompson [2] observed that the surface temperature of a slab taken immediately after a sudden thermal shock is 300 K higher than that predicted by Fourier's law. The disagreement between Fourier prediction and such experimental observation is rooted in the unrealistic propagation speed of thermal signal adopted by Fourier's law. In reality, a thermal signal travels at a finite speed, making a thermal response to behave like a wave. Such wave-like behavior was first experimentally observed in solid He^4 by Ackerman [3]. To better describe this wave-like behavior, instead of using Fourier's law, the Maxwell–Cattaneo equation, which takes finite thermal signal traveling speed into account, can be used. This, however, leads to a hyperbolic governing equation on heat conduction. The hyperbolic equation is more difficult to solve both theoretically and numerically than its parabolic counterpart produced by Fourier's law. A popular numerical method to solve hyperbolic heat conduction equation is using Laplace operator to transform the first and second-order derivatives in the time domain to second-degree polynomials in the Laplace domain, allowing the governing equation to be easily solved [4,5]. An issue of this approach is that the solution has to be inversely transformed from the Laplace domain back to the time domain by using Laplace inverse transform, which is a complicated and tedious process. As a result, most published work using Laplace transform to solve hyperbolic heat conduction is limited to one dimensional problems [6,7], and there have been very few two- or three-dimensional studies [8]. Another numerical approach is using finite differencing on the first and second-order derivatives in the time domain [9]. These derivatives can be discretized using backward or central difference techniques, and the processes involved are much simpler than those in Laplace inverse transform. Yet, there are different issues associated with this approach. Central difference is more accurate but is unstable, prone to introduce unrealistic oscillation to the solution. Backward difference, on the other hand, is more stable but less accurate, requiring the use of very small time step to achieve satisfactory accuracy [9]. Despite these issues, a crucial advantage of this approach over Laplace transform is that it can be easily applied to solve two- or even three-dimensional hyperbolic heat conduction problems. Given that most engineering problems are multi-dimensional in nature and often involve complicated geometries, to numerically solve hyperbolic equation by finite differencing is a more practical approach.

Quantitative studies of heat transfer processes occurring in many industrial applications often require accurate knowledge of boundary conditions, such as heat flux, or thermophysical properties of the materials involved. These important quantities were conventionally obtained

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Nomenclature

C_p	specific heat (J kg^{-1} K^{-1})
c	propagation speed of thermal wave (m s^{-1})
H	dimensionless convection heat transfer coefficient function at the lateral surface
H_t	dimensionless convection heat transfer coefficient function at the tip surface
h	convection heat transfer coefficient at the lateral surface (W m^{-2} K^{-1})
h_t	convection heat transfer coefficient at the tip surface (W m^{-2} K^{-1})
J	functional
J'	gradient of functional
k	thermal conductivity (W m^{-1} K^{-1})
L	pin fin length (m)
p	direction of descent
Q	dimensionless base heat flux
q	base heat flux (W m^{-2})
R	radius of pin fin (m)
r	space coordinate in r-direction (m)
T	temperature (K)
T_r	reference temperature (K)
T_∞	ambient temperature (K)
t	time (s)
x	space coordinate in x-direction (m)
x_m	temperature measurement position in x-direction (m)

Greek symbols

Δ	small variation quality
α	thermal diffusivity, $k/\rho C_p$ (m^2 s^{-1})
β	step size
γ	conjugate coefficient
ε	very small value
ς	dimensionless space coordinate in r-direction
η	dimensionless space coordinate in x-direction
θ	dimensionless temperature
λ	variable used in the adjoint problem
μ	transformed dimensionless time, $\xi_f - \xi$
ξ	dimensionless time
ξ_f	dimensionless final time of the measurement
ρ	density (kg m^{-3})
σ	standard deviation
τ	relaxation time (s)
ϖ	random variable

Superscript/subscript

K	iterative number

by expensive experimental methods which normally involve delicate and sophisticated equipments. In recent years, however, the studies of inverse heat conduction problem (IHCP) have offered convenient alternatives, which largely scale down experimental work, to obtain accurate thermophysical quantities such as heat sources, material's thermal properties, and boundary temperature or heat flux distributions, in many heat conduction problems. For example, Chen and Su [10] provided an inverse analysis to estimate the boundary thermal behavior of a furnace with two layer walls. The unknown temperature distribution of the outer surface and the geometry of the inner surface are estimated from the temperatures of a small number of measured points within the furnace wall. Chen et al. [11] solved the inverse problem to estimate the inlet jet temperature in an impinging jet cooling problem. Given the maximum allowable plate temperature and the extent of the area on plate where temperature needs to be controlled, the jet temperature required to meet the two demands can be determined. Yang and Chen [12] adopted an inverse algorithm based on the conjugate gradient method (CGM) and the discrepancy principle to estimate the unknown space- and time-dependent heat flux of the disc in a nonlinear disc brake system from the knowledge of temperature measurements taken within the disc. In the above cases, the direct heat conduction problems are concerned with the determination of temperature at interior points of a region when the initial and boundary conditions, heat generation, and material properties are specified, whereas, the IHCP involves the determination of the surface conditions, energy generation, thermophysical properties, etc., from the knowledge of temperature measurements taken within the body. To date, a variety of analytical and numerical techniques have been developed for the solution of the inverse heat conduction problems, for example, the conjugate gradient method [13–15], the genetic algorithm [16], and the linear least-squares error method [17], etc.

Although there have been a great number of reports dealing with the inverse solutions of classical Fourier heat conduction problems, however, the study on inverse non-Fourier heat conduction problem is much limited in the literature. For example, Chen et al. [18] applied the least-square scheme in conjunction with the hyperbolic shape function, the control volume method, and the Laplace transform technique to estimate the unknown surface conditions of one-dimensional hyperbolic inverse heat conduction problems. Huang and Wu [19] studied the inverse non-Fourier problem of a straight fin by an iterative regularization method in estimating the unknown base temperature based on the boundary temperature measurements. Yang [20] proposed a sequential method for estimating the boundary conditions in a two-dimensional hyperbolic heat conduction problem. The inverse solution is deduced from a finite-difference method, the concept of future time, and a modified Newton–Raphson method. Das et al. [21] estimated the extinction coefficient and the conduction–radiation parameter simultaneously in a non-Fourier conduction and radiation heat transfer problem. The problem is solved using the genetic algorithm in combination with the lattice Boltzmann method and the finite-volume method. The effects of measurement errors and genetic parameters on the accuracies of the estimated parameters are also investigated.

Pin fins have been widely used to enhance the heat transfer rate in many engineering applications, it is important to have knowledge about the subjected base heat flux of the fins if the performance of them is to be properly evaluated. The modeling of the heat transfer process of fins can reduce experimental cost and shed light into the heat transfer process. Therefore, the focus of the present study is to develop an inverse hyperbolic analysis for estimating the unknown space- and time-dependent heat flux at the base of a pin fin from the knowledge of temperature measurements taken within the medium. The non-Fourier effect is considered in the formulation of heat conduction equation. An analysis of this kind poses significant implications on several industrial applications such as laser heating, pressure vessels and pipes, chemical plants, etc. To this end, we present the conjugate gradient method and the discrepancy principle [22] to estimate the unknown space- and time-dependent heat flux by using the simulated temperature measurements. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem, which will be discussed in detail in the following sections. Here, the first- and second-derivatives in all governing equations in the three problems are discretized by backward differencing to avoid adding the tedious Laplace inverse transform on top of this already complicated inverse process.