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Analysis of turbulent double-diffusive free convection in porous media using the two-energy equation model ☆

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ARTICLE INFO

ABSTRACT

This paper presents an analysis of macroscopic heat and mass transport for turbulent flow in permeable structures, which is based on the thermal non-equilibrium assumption between the porous matrix and the working fluid. Two driving mechanisms are here considered to contribute to the overall momentum transport, namely fluid-temperature driven and concentration driven mass fluxes. The fluid temperature, however, is also affected by the solid temperature distribution as the two phases exchange heat through their interfacial area. Essentially, here the double-diffusive natural convection mechanism is investigated for the fluid phase in turbulent regime. Equations are presented based on the double-decomposition concept, which considers both time fluctuations and spatial deviations about mean values. This work intends to demonstrate that additional transport mechanisms are mathematically derived if velocity, fluid temperature and mass concentration simultaneously present time fluctuations and spatial deviations about average values. A modeled form for the entire set of transport equations is presented where turbulent transfer is based on a macroscopic version of the k–ε model.

1. Introduction

Analyses of double-diffusive phenomena in free convection in permeable media has many environmental and industrial applications, such as in oil and gas extraction, movement of gas concentration into the ground, contaminant dispersion in soils, grain storage and drying, petrochemical processes, electrochemical processes, to mention a few [1–9]. In some specific applications, the voids are large enough and the fluid mixture may become turbulent. In such instances, difficulties arise in the proper mathematical modeling of the transport processes under both temperature and concentration gradients.

Usually, modeling of macroscopic transport for incompressible flows in rigid porous media has been based on the volume-average methodology for either heat or mass transfer [10–14]. If fluctuations in time are also of concern due the existence of turbulence in the intra-pore space, a variety of mathematical models have been published in the literature in the last decade. One of such views, which entails simultaneous application of both time and volume averaging operators to all governing equations, has been organized and published in a book [15] that describes, in detail, an idea known in the literature as the double-decomposition concept (see chapter 3, pgs. 27–32 in ref. [15] for details).

In an earlier work [16], double-diffusive effects in porous media have been treated considering thermal equilibrium between the porous matrix and the permeating fluid. Or say, in ref. [16] the fluid temperate was assumed to be the same of that of the solid when analyzing double-diffusive mechanisms. Later [17], buoyancy-free flows were investigated

with the so-called two-energy-equation model, or 2EEM for short, which is based on the Local Thermal Non-equilibrium Hypothesis (LTNE) meaning that the average temperature of the fluid is not equal to the average temperature of the solid matrix However, in ref. [17] no double-diffusion was considered.

Therefore, the purpose of this contribution is to extend the work in ref. [16] on turbulent double-diffusion using only one energy equation, assuming now the thermal non-equilibrium hypotheses in ref. [17], which requires an independent energy balance for each phase. As such, the expectation herein is that, by combining now such two models that were developed on separate, a larger number of physical processes can now be more realistically tackled.

2. Local instantaneous transport equation

The steady-state local (microscopic) instantaneous transport equations for an incompressible binary fluid mixture with constant properties flowing in an inert heterogeneous medium are given in details elsewhere and for that, they will be just repeated here. They read:

within the fluid:

$$\text{Continuity} \quad \nabla \cdot \mathbf{u} = 0 \tag{1}$$

$$\text{Momentum} \quad \rho \nabla \cdot (\mathbf{uu}) = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{g} \tag{2}$$

$$\text{Energy-fluid phase} \left(\rho c_p\right)_f \left\{ \nabla \cdot \left(\mathbf{u} T_f\right) \right\} = \nabla \cdot \left(k_f \nabla T_f\right) + S_f. \tag{3}$$

$$\text{Mass concentration} \quad \rho \nabla \cdot (\mathbf{u}\, m_\ell + \mathbf{J}_\ell) = \rho R_\ell \tag{4}$$

Nomenclature

c_F	Forchheimer coefficient
C_ℓ	volumetric molar concentration
c_p	specific heat
D_ℓ	diffusion coefficient
$\mathbf{D}_{disp}$	Mass dispersion
$\mathbf{D}_{disp,t}$	turbulent mass dispersion
$\mathbf{D}_t$	turbulent mass flux
$\mathbf{g}$	gravity acceleration vector
$\mathbf{I}$	unity tensor
$\mathbf{J}_\ell$	mass diffusion coefficient
k	turbulent kinetic energy per unit mass, $k = \overline{\mathbf{u'} \cdot \mathbf{u'}}/2$
$<k>^i$	intrinsic (fluid) average of k
K	permeability
ℓ	chemical species
m_ℓ	mass fraction of component ℓ
M_ℓ	molar weight of component ℓ
p	pressure
Pr_t	turbulent Prandtl number
Sc_t	turbulent Schmidt number
T	temperature
$\mathbf{u}$	mass-averaged velocity of the mixture
$\overline{\mathbf{u}}_D$	Darcy velocity vector
$\mathbf{u}_\ell$	velocity of species ℓ

Greek symbols

β	thermal expansion coefficient
β_C	salute expansion coefficient
$\beta\phi$	macroscopic thermal expansion coefficient
$\beta_{C\phi}$	macroscopic salute expansion coefficient
λ	fluid thermal conductivity
μ	fluid mixture viscosity
μ_t	Turbulent viscosity
$\mu_{t\phi}$	macroscopic turbulent viscosity
ε	dissipation rate of k
$\langle\varepsilon\rangle^i$	intrinsic (fluid) average of ε
ρ	bulk density of the mixture
ρ_ℓ	mass density of species ℓ
ϕ	porosity

Subscripts

β	buoyancy
ℓ	chemical species
t	turbulent
ϕ	macroscopic
C	concentration

Superscripts

i	intrinsic (fluid) average
v	volume (fluid + solid) average
k	turbulent kinetic energy
s,f	solid, fluid

within the solid:

Energy-solid shase (porous matrix) $0 = \nabla \cdot (k_s \nabla T_s) + S_s$. (5)

where $\mathbf{u}$ is the mass-averaged velocity of the mixture, $\mathbf{u} = \sum m_\ell \mathbf{u}_\ell$, $\mathbf{u}_\ell$ is the velocity of species ℓ, m_ℓ is the mass fraction of component ℓ,

defined as $m_\ell = \rho_\ell/\rho$, ρ_ℓ is the mass density of species ℓ (mass of ℓ over total mixture volume), ρ is the bulk density of the mixture $\left(\rho = \sum_\ell \rho_\ell\right)$, p is the pressure, μ is the fluid mixture viscosity, $\mathbf{g}$ is the gravity acceleration vector, c_p is the specific heat, the subscripts f and s refer to fluid and solid phases, respectively, T_f and T_s are the fluid and solid temperature, k_f and k_s are the fluid and solid thermal conductivities and S is the heat generation term. If there is no heat generation either in the solid or in the fluid, one has further $S_f = S_s = 0$. The generation rate of species ℓ per unit of mixture mass is given in Eq. (4) by R_ℓ. Also, as pointed out in ref. [16], an alternative way of writing the mass transport equation is using the volumetric molar concentration C_ℓ (mol of ℓ over total mixture volume), the molar weight M_ℓ (g/mol of ℓ) and the molar generation/destruction rate R_ℓ^* (mol of ℓ/ total mixture volume), giving:

$$M_\ell \nabla \cdot (\mathbf{u} \, C_\ell + \mathbf{J}_\ell) = M_\ell R_\ell^*.$$ (6)

Further, the mass diffusion flux $\mathbf{J}_\ell$ (mass of ℓ per unit area per unit time) in Eq. (4) or (6) is due to the velocity slip of species ℓ,

$$\mathbf{J} = \rho_\ell(\mathbf{u}_\ell - \mathbf{u}) = -\rho_\ell D_\ell \nabla m_\ell = -M_\ell D_\ell \nabla C_\ell$$ (7)

where D_ℓ is the diffusion coefficient of species ℓ into the mixture. The second equality in Eq. (7) is known as Fick's Law, which is a constitutive equation strictly valid for binary mixtures under the absence of any additional driving mechanisms for mass transfer [10]. Therefore, no *Soret* or *Dufour* effects are here considered.

Rearranging Eq. (6) for an inert species, dividing it by M_ℓ and dropping the index ℓ for a simple binary mixture, one has,

$$\nabla \cdot (\mathbf{u} \, C) = \nabla \cdot (D\nabla C).$$ (8)

If one considers that the density in the last term of Eq. (2) varies with fluid temperature and concentration, for natural convection flow, the Boussinesq hypothesis reads, after renaming this density ρ_T,

$$\rho_T \cong \rho \left[1 - \beta\left(T_f - T_{ref}\right) - \beta_C\left(C - C_{ref}\right)\right]$$ (9)

where the subscript *ref* indicates a reference value and β and β_C are the thermal and salute expansion coefficients, respectively, defined by,

$$\beta = -\frac{1}{\rho}\frac{\partial\rho}{\partial T_f}\bigg|_{p,C}, \beta_C = -\frac{1}{\rho}\frac{\partial\rho}{\partial C}\bigg|_{p,T_f}.$$ (10)

Here, it is interesting to point out that in ref. [16] the temperature used in Eq. (9) was the same as that of the solid, $T = T_f = T_s$. Further, it is important to note that, as it is going to be shown below, after volume averaging Eqs. (3) and (5), T_f is going to be related to T_s due to the exchange of heat between the two phases across the interstitial area. Also, Eq. (9) is an approximation of Eq. (10) and shows how density varies with the fluid temperature and mass concentration in the body force term of the momentum equation. Substituting now Eq. (9) into Eq. (2), one has,

$$\rho\nabla \cdot (\mathbf{uu}) = -\nabla p + \mu\nabla^2\mathbf{u} + \rho\mathbf{g}\left[1 - \beta\left(T_f - T_{ref}\right) - \beta_C\left(C - C_{ref}\right)\right].$$ (11)

Thus, the momentum equation becomes after some rearrangement,

$$\rho\nabla \cdot (\mathbf{uu}) = -(\nabla p)^* + \mu\nabla^2\mathbf{u} - \rho\mathbf{g}\left[\left(\beta\left(T_f - T_{ref}\right) + \beta_C\left(C - C_{ref}\right)\right)\right]$$ (12)

where $(\nabla p)^* = \nabla p - \rho\mathbf{g}$ is a modified pressure gradient.