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Theoretical analysis of wall thermal inertial effects on heat transfer of pulsating laminar flow in a channel☆

Hongsheng Yuan, Sichao Tan *, Nailiang Zhuang, Linghong Tang

Fundamental Science on Nuclear Safety and Simulation Technology Laboratory, Harbin Engineering University, Heilongjiang 150001, PR China

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ABSTRACT

The effects of wall heat capacity on heat transfer of pulsating laminar flow with constant heat input between two parallel plates are theoretically investigated. The analytical solution of the fully developed thermal profile under constant wall heat flux is obtained. The results show that both the fluid temperature and Nusselt number fluctuate periodically as the driven pressure pulsates. The effects of the wall thermal inertia and Prandtl number, as well as pulsation amplitude and frequency on the fluctuations are investigated. Compared with pulsating flow without considering wall thermal inertia, the fluctuation amplitude of the fluid temperature and Nusselt number is smaller while the mean temperature difference is larger. The average Nusselt number, for both considering and neglecting wall thermal inertia, is reduced by pulsating flow. The reduction is up to 10% and cannot be neglected.

1. Introduction

Pulsating flow, as a common phenomenon in both natural and engineering systems, has received extensive attention. As for velocity distribution of pulsating flow, theoretical analysis is in accordance with both experimental data and numerical studies [1–4]. However, the available experimental data [5–11] in laminar pulsating flow showed that pulsation could decrease or increase the Nusselt number. Analytical studies [12–14] in fully developed region found that the Nusselt number fluctuates periodically around the value of steady flow. In studies [16,17], a higher heat transfer rate was obtained. Guo and Sung [18] pointed out that different definitions of the average Nusselt number will lead to contradictory conclusions. However, Krishnan and Satri [15] and Hemida et al. [19] found that heat transfer is weakened but is not of much practical importance. Simulation results [19] showed that wall thermal resistance capacity can damp out pulsation effects in thermally developing region.

Based on the review of current literatures, it is found that studies on pulsating laminar heat transfer led to contradictory results and are often confined to constant wall temperature and constant heat flux conditions. Researches on the effects of wall heat capacity on pulsating flow are limited. However, wall thermal inertia is commonly encountered in engineering systems. It is to be expected that the channel wall can absorb and release heat, and then the temperature pulsation and thermal stresses oscillation across the wall are reduced. Such consequences have positive effects on the thermodynamic and safety performance of the facilities. Hence we theoretically investigate pulsating heat transfer considering the effects of wall heat capacity in this work.

2. Theoretical analysis

The analysis concerns laminar and incompressible flow with both fully developed velocity and temperature profile. Second effects such as viscous dissipation and variation of thermophysical properties are neglected for simplification. The velocity and temperature profiles are obtained by solving governing equations in this Section.

2.1. Velocity of pulsating flow

For pulsating laminar flow between two parallel plates, the momentum equation and boundary conditions can be expressed as

$$\begin{cases} \dfrac{\partial u}{\partial t} = -\dfrac{1}{\rho}\left(\dfrac{\partial p}{\partial x}\right)_s \left[1 + \dfrac{\gamma}{2}\cos(\omega t)\right] + \nu \dfrac{\partial^2 u}{\partial y^2} \\ y = \pm d : u = 0 \end{cases} \tag{1}$$

where $\left(\frac{\partial p}{\partial x}\right)_s$ is the steady part of pressure gradient, γ represents relative oscillation amplitude of pressure gradient, and ω denotes pulsatile frequency d is half the width between parallel plates.

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* Corresponding author at: College of Nuclear Science and Technology, Harbin Engineering University, Heilongjiang 150001, PR China.
E-mail address: tansichao@hrbeu.edu.cn (S. Tan).

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Nomenclature

C	constant
c	specific heat at constant pressure
d	half width between parallel plates
e	Euler number
i	imaginary unit
k	thermal conductivity
Nu	Nusselt number
Pr	Prandtl number of fluid
p	pressure
q	heat flux
T	temperature
t	time
u	velocity
x, y	dimensional coordinates
X	$4x$ / $(dRePr)$

Greek letters

ρ	density
γ	half oscillation amplitude of pressure
δ	wall thickness
Θ	dimensionless temperature
μ	dynamic viscosity;
ν	kinematic viscosity
ω	oscillation frequency

Subscripts

b	bulk
s	non-pulsating component
t	time-dependent component
w	wall

Superscripts

*	dimensionless or normalized quantity
-	average quantity

In order to transform the above system into a non-dimensional form, the following transformations will be applied:

$$y* = \frac{y}{d}, \omega* = \frac{\omega d^2}{\nu}, t* = \frac{\nu t}{d^2}, u_{\mathrm{m}} = -\frac{d^2}{3\mu}\left(\frac{\partial p}{\partial x}\right)_{\mathrm{s}}, u* = \frac{u}{u_{\mathrm{m}}}.$$

Since Eq. (1) is a linear system, we can separate the velocity into the steady part u^*_{s} and pulsating part u^*_{t}. By the method of separation of variables, the flow field is obtained:

$$u* = \frac{3}{2}\left(1-y*^2\right) + \frac{3\gamma i}{2\omega*}\left(\frac{\cos\sqrt{i\omega*}y*}{\cos\sqrt{i\omega*}} - 1\right)e^{-i\omega* t*} \tag{2}$$

where i is the imaginary factor.

2.2. Temperature of pulsating flow

For laminar flow between two parallel palates, the energy equation can be simplified to

$$\rho c\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) - k\frac{\partial^2 T}{\partial y^2} = 0. \tag{3}$$

Due to the high thermal conductivity of the metal wall, wall temperature will be assumed constant along the wall thickness δ, but variable along the pipe axis as the first approximation. In this case, the boundary conditions of fully developed flow would be in the following form.

$$\begin{cases} y = 0 : \dfrac{\partial T}{\partial y} = 0 \\ y = d : k\dfrac{\partial T}{\partial y} + \rho_{\mathrm{w}} c_{\mathrm{w}} \delta \dfrac{\partial T}{\partial t} = q \end{cases} \tag{4}$$

where ρ_{w} is the wall density, c_{w} is the wall specific heat, q is the heat input.

To get dimensionless equations, the following dimensionless parameters are introduced.

$$\Theta = \frac{T - T_{\mathrm{s,m}}}{qd/k}, x* = \frac{x}{d}, X = \frac{4x*}{RePr}, Re = \frac{4u_{\mathrm{m}} d}{\nu}, c*_{\mathrm{A}} = c * Pr,$$
$$c* = \frac{\rho_{\mathrm{w}} c_{\mathrm{w}} \delta}{\rho c d}$$

where $T_{s,m}$ is the inlet bulk temperature of fully developed pulsating laminar flow, and c^* is the ratio of wall specific heat to fluid specific heat.

The dimensionless energy equation and boundary conditions take the form:

$$\begin{cases} Pr\dfrac{\partial \Theta}{\partial t*} + u * \dfrac{\partial \Theta}{\partial X} = \dfrac{\partial^2 \Theta}{\partial y*^2} \\ y = 0 : \dfrac{\partial \Theta}{\partial y*} = 0 \\ y = d : \dfrac{\partial \Theta}{\partial y*} + c*_A \dfrac{\partial \Theta}{\partial t*} = 1. \end{cases} \tag{5}$$

In order to obtain an analytical solution of the fully developed temperature profile, the dimensionless temperature can be split into two parts:

$$\Theta = \Theta_{\mathrm{s}}(y*, X) + \gamma \Theta_{\mathrm{t}}(y*, t*) \tag{6}$$

where Θ_{s} represents the steady part of the temperature, while Θ_{t} represents variation in temperature due to the oscillatory part of the pulsating flow.

Yu [13] has solved the time average system and got a correlation for the steady state temperature.

$$\Theta_{\mathrm{s}} = X + \frac{3}{2}\left(\frac{y*^2}{2} - \frac{y*^4}{12}\right) - \frac{39}{280} \tag{7}$$

By the method of separation of variables, the transient temperature can be obtained.

$$\Theta_{\mathrm{t}} = \begin{cases} \left[C_1 \cos\left(\sqrt{i\omega*} y*\right) - \dfrac{3}{2\omega*^2} + \dfrac{3i}{4\omega * \sqrt{i\omega*}} \dfrac{y * \sin\left(\sqrt{i\omega*} y*\right)}{\cos\left(\sqrt{i\omega*}\right)}\right] e^{-i\omega* t*}, Pr = 1 \\ \left[C_1' \cos\left(\sqrt{iPr\omega*} y*\right) - \dfrac{3}{2Pr\omega*^2} + \dfrac{3}{2\omega*^2 (Pr-1)} \dfrac{\cos\left(\sqrt{i\omega*} y*\right)}{\cos\left(\sqrt{i\omega*}\right)}\right] e^{-i\omega* t*}, Pr \neq 1 \end{cases} \tag{8}$$

where

$$C_1 = \frac{3i\left(\sqrt{i\omega*} + 2c_A * \sqrt{i\omega*} + \tan\sqrt{i\omega*} - ic_A * \omega \tan\sqrt{i\omega*}\right)}{4\sqrt{i\omega*}\omega * \left(ic_A * \omega * \cos\sqrt{i\omega*} + \sqrt{i\omega*} \sin\sqrt{i\omega*}\right)}$$
$$C_1' = -\frac{3\left(c_A * \omega * - iPr\sqrt{i\omega*} \tan\sqrt{i\omega*}\right)}{2Pr\omega*^2 (Pr-1)\left(c_A * \omega * \cos\sqrt{iPr\omega*} - i\sqrt{iPr\omega*} \sin\sqrt{iPr\omega*}\right)}.$$