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Effects of magnetic field and thermal radiation on stagnation flow and heat transfer of nanofluid over a shrinking surface ☆

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ABSTRACT

In this paper, the problem of steady two-dimensional magnetohydrodynamic (MHD) stagnation-point flow and heat transfer, with thermal radiation, of a nanofluid past a shrinking sheet is investigated numerically. Both the effects of Brownian motion and thermophoresis are considered simultaneously. A similarity transformation is used to transform the governing partial differential equations to a system of nonlinear ordinary differential equations which are solved numerically using a shooting technique. A similarity solution is presented which depends on the magnetic parameter (M), radiation parameter (R), Brownian motion number (Nb), thermophoresis number (Nt), Prandtl number (Pr), Lewis number (Le) and the ratio of the rate constants of the shrinking velocity to the free stream velocity (α). Interesting solution behavior is observed with multiple solution branches for certain parameter domain. The results of the present paper show that the velocity, temperature, the wall shear stress, the Nusselt number and the Sherwood number are strongly influenced by the magnetic parameter. A comparative study between the previously published results and the present results for a special case is found to be in good agreement.

1. Introduction

Boundary layer flow behavior over a stretching surface is important as it occurs in several engineering processes, for example, materials manufactured by extrusion, annealing and tinning of copper wires, glass blowing, continuous cooling and fiber spinning and many others. During the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. In these cases, the final product of desired characteristic depends on the rate of cooling and the process of stretching. In view of these applications, Sakiadis [1] investigated the boundary layer flow of a viscous fluid past a moving solid surface. Some more representative studies in this direction were presented through their works. (see [2–5]).

Stagnation flow, describing the fluid motion near the stagnation region, exists on all solid bodies moving in a fluid. The stagnation region encounters the highest pressure, the highest heat transfer and the highest rate of mass decomposition. The stagnation point flows arise in many applications including flows over the tips of rockets, aircrafts, submarines and oil ships. Chiam [6] was the first to study the steady stagnation-point flow over an elastic surface considering the equal values of the stretching and free stream velocities. Later Mahapatra and Gupta [7] reconsidered this flow problem by choosing different stretching and free stream velocities and they ensured the existence of boundary layers. A large number of analytical and numerical studies explaining various aspects of the boundary layer stagnation flow over a stretching surface are available. Mention may be made to some interesting works (see [8–10]).

Heat transfer is an important process in Physics and Engineering, and consequently improvements in heat transfer characteristics will improve the efficiency of many processes. A nanofluid is a new class of heat transfer fluids that contains a base fluid and nanoparticles. The use of additives is a technique applied to enhance the heat transfer performance of base fluids. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquid [11]. Thus nanofluids have many applications in industry such as coolants, lubricants, heat exchangers, micro channel heat sinks and many others.

The term ‘nanofluid’ was first proposed by Choi [12] to indicate engineered colloids composed of nanoparticles dispersed in a base fluid. A comprehensive survey of convective transport in nanofluids was made by Buongiorna [13] who considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid. Among these mechanisms, only Brownian diffusion and thermophoresis were found to be important. Later the influence of

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nanoparticles on natural convection boundary layer flow over a vertical plate was considered by Kuznetsov and Nield [14]. The paper Khan and Pop [15] was the first which considered the problem of boundary layer flow past a stretching sheet in a nanofluid and later Bachok et al. [16] investigated the boundary layer flow of a nanofluid over a moving surface in a flowing fluid. Since then many researchers have investigated the similar problem in a nanofluid with various physical aspects (see [17–22]).

For the flow over a shrinking sheet, the fluid is attracted towards a slot and as a result it shows quite different characteristics from the stretching case. From a physical point of view, vorticity generated at the shrinking sheet is not confined within a boundary layer and a steady flow is not possible unless either a stagnation flow is applied or adequate suction is applied at the sheet. As discussed by Goldstein [23], this new type of shrinking flow is essentially a backward flow. Miklavcic and Wang [24] were the first who have investigated the flow over a shrinking sheet with suction effect. Steady two-dimensional and axisymmetric boundary layer stagnation point flow and heat transfer towards a shrinking sheet was analyzed by Wang [25]. The existence and uniqueness results for MHD stagnation point flow over a stretching/shrinking sheet were considered by Van Gorder et al. [26]. Thereafter various aspects of stagnation point flow and heat transfer over a shrinking sheet were investigated by many authors (see [27–32]).

All studies mentioned above refer to the stagnation point flow towards a stretching/shrinking sheet in a viscous and Newtonian fluid. Bachok et al. [33] investigated the effects of solid volume fraction and the type of the nanoparticles on the fluid flow and heat transfer characteristics of a nanofluid over a shrinking sheet. To the author's knowledge no studies have thus far been communicated with regard to MHD boundary layer stagnation flow and heat transfer of a nanofluid past a shrinking sheet. The objective of the present paper is therefore to extend the work of Khan and Pop [15] by taking MHD flow over a shrinking sheet. The effects of magnetic field parameter (M), Brownian motion parameter (Nb), thermophoresis number (Nt), Prandtl number (Pr), Lewis number (Le), radiation parameter (R) and the velocity ratio parameter α on the relevant flow variables are described in detail. The present study is of immediate interest to all those processes which are highly affected with heat enhancement concept e.g. cooling of metallic sheets or electronic chips etc.

2. Flow analysis

Consider the steady two-dimensional MHD stagnation-point flow of an incompressible viscous electrically conducting nanofluid impinging normally on a shrinking sheet. The fluid is subjected to a uniform transverse magnetic field of strength B_0. Fig. 1 describes the physical model

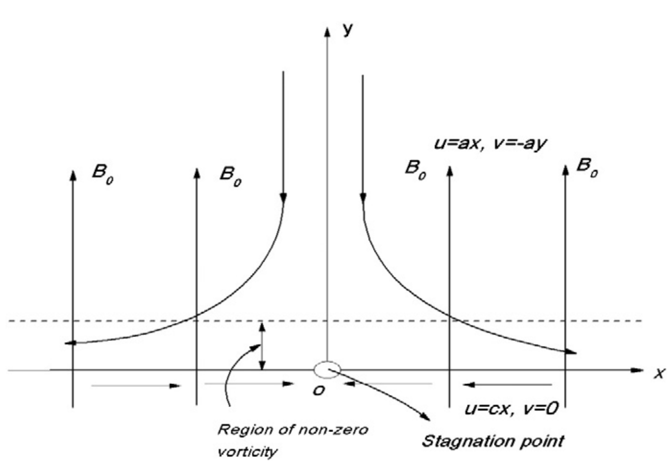

Fig. 1. Physical model and the coordinate system.

Table 1
Comparison of the values of $F''(0)$ (with M = 0 for viscous fluid) for stretching sheet with different values of α (> 0).

α	Present study	Wang [25]	Lok et al. [30]
0.0	1.232588	1.232588	1.232588
0.1	1.146560	1.146560	1.146561
0.2	1.051131	1.051130	1.051130
0.5	0.713296	0.713300	0.713295
1.0	0.000000	0.000000	0.000000
2.0	−1.887308	−1.88731	−1.887307
5.0	−10.264751	−10.26475	−10.264749

and the coordinate system, where the x and y axes are measured along the surface of the sheet and normal to it, respectively. It is assumed that the velocity of the stretching/shrinking sheet is $u_w(x) = cx$ and the velocity outside the boundary layer is $U(x) = ax$ where a and c are constants with $a > 0$. We note that $c > 0$ and $c < 0$ correspond to stretching and shrinking sheets, respectively.

The basic steady conservation of mass, momentum, thermal energy and nanoparticles equations for nanofluids can be written in Cartesian coordinates x and y as, (see [15])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\sigma {B_0}^2}{\rho_f}(U-u) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho_f c_p}\frac{\partial q_r}{\partial y} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

In writing Eq. (2), we have neglected the induced magnetic field since the magnetic Reynolds number for the flow is assumed to be very small. This assumption is justified for flow of electrically conducting fluids such as liquid metals e.g., mercury, liquid sodium etc. (Shercliff [34]). Here u and v are the velocity components along the x and y directions, respectively, $U(x)$ is the free stream velocity, T is the fluid temperature and C is the nanoparticle volume fraction, σ is the electrical conductivity of the fluid, ν is the kinematic viscosity, α_m is the thermal diffusivity, ρ_f is the density of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoresis diffusion coefficient, c_p is the specific heat at constant pressure, q_r is the radiative heat flux and τ is the ratio of the effective heat capacity of the nanoparticle material to the heat capacity of the ordinary fluid. It is assumed that the wall temperature T_w and the nanoparticle volume fraction C_w are constant at the surface. Also when y tends to infinity, the ambient values of the temperature and the nanoparticle volume fraction attain to constant values of T_∞ and C_∞, respectively.

The boundary conditions for the problem are

$$\begin{aligned} &u = u_w(x) = cx, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \\ &u \to U(x) = ax, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{5}$$

The radiative heat flux q_r is described by Rosseland approximation (see [35]) such that

$$q_r = -\frac{4\delta}{3k_1}\frac{\partial T^4}{\partial y} \tag{6}$$

where δ is the Stefan–Boltzmann constant and k_1 is the mean absorption