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Free convection along a convectively heated vertical flat sheet embedded in a saturated porous medium☆

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ABSTRACT

An investigation is made for steady free convection about a vertical flat plate embedded in a saturated porous medium due to a convectively heated wall. With the boundary layer theory, the governing equations describing the momentum and energy conservations are reduced to a couple of ordinary differential equations with convective boundary conditions. The exact solutions are then obtained analytically. The velocity and temperature distributions, as well as the local Nusselt number are presented and analyzed. The current analysis is then applied to the convective heat transfer about a dike intruded in an aquifer and the relevant physical quantities are calculated and discussed.

1. Introduction

Investigations of natural convection heat transfer from a vertical plate embedded in various fluids have been done by many researchers owing to their important applications in many engineering and industrial progresses, such as the storage of radioactive nuclear materials, ground water pollution, insulation of building and cold storage, drying processes, transpiration cooling, powder metallurgy, agriculture engineering and so on. Among these works, Cheng and Minkowycz [1] made an analysis on the free convection about a vertical flat plate embedded in a saturated porous medium. Their results were found to be very accurate for large Rayleigh numbers. They then applied their analysis to convective heat transfer about an isothermal dike intruded in an aquifer. Cheng and Minkowycz's problem [1] was included into the book by Bejan [2] as an exemplificative configuration and solution to illustrate. Cheng and Minkowycz's problem [1] was extended to various cases of heat and mass transfer by Bejan and Khair [3], Nield and Bejan [4], Chamkha and Quadri [5], and Chamkha and Pop [6]. The extension of Cheng and Minkowycz's problem [1] to nanofluids was made by Nield and Kuznetsov [7,8], etc.

Recently, Aziz [9] investigates the classical problem of hydrodynamic and thermal boundary layers over a flat plate in a uniform stream of fluid with a convective surface boundary condition in the framework of the boundary layer approximations. Aziz [10] then considered the hydrodynamic and thermal slip flow boundary layers over a flat surface with constant heat flux boundary condition. Ishak [11] provided the similarity solutions for flow and heat transfer over a permeable surface with convective boundary condition. Makinde and Aziz [12] and Makinde [13,14] investigated the buoyancy effects on thermal boundary layer over a vertical plate subject a convective surface boundary condition. Hayat [15] analyzed the steady flow of an Eyring Powell fluid over a moving surface with convective boundary conditions using the homotopy analysis method (HAM) [16–20]. Very recently Lok et al. [21] and Merkin et al. [22] have studied the steady mixed convection flow past a vertical flat plate embedded in a porous medium subject to a convective boundary condition. However, the present problem refers to the case of a free convection with a convective boundary conditions using HAM technique.

The purpose of the present work is to study the steady free convection past a vertical flat plate embedded in a saturated porous medium with a convective boundary condition and to apply it to convective heat transfer about a dike intruded in an aquifer. It is found that when the convective heat transfer coefficient of the wall is proportional to $x^{-1/2}$, similarity solutions can be obtained via a set of similarity transformations. The reduced governing equations describing the momentum and energy conservations are then formulated and solved analytically using the optimal HAM technique (OHAM) [23]. The velocity and temperature distribution, as well as the boundary layer thickness, and the Nusselt number are presented and discussed.

2. Mathematical formulation

Consider the steady free convection about a vertical flat sheet embedded in a saturated porous medium. As shown in Fig. 1, the fluid is

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Nomenclature

T_f	fluid temperature in the left side
T_∞	fluid temperature in the right side
x	dimensional axis being measured along the plate
y	dimensional axis being normal to the plate
u	Darcy's velocity in the x directions
v	Darcy's velocity in the y directions
p	pressure
T	temperature
K	permeability of the saturated porous medium
g	acceleration due to gravity
k_m	thermal conductivity of the saturated porous medium
C_p	specific heat of the fluid
$h_f(x)$	heat transfer coefficient due to fluid temperature
Ra_x	modified local Rayleigh number in a porous medium
η	dimensionless variable
f	dimensionless function dependent in terms of stream function
E_k	error function
Nu_x	local Nusselt number
$q_w(x)$	local surface heat flux through the wall
Q	overall surface heat transfer rate for a flat plate
S	span dimension of the sheet
$\overline{Nu}$	average Nusselt number
$\overline{h}$	average heat transfer coefficient
Ra	Rayleigh number
μ	dynamic viscosity of the fluid
ρ	density of the fluid
β	thermal expansion coefficient of the fluid
ρ_∞	density of the ambient fluid
α	equivalent thermal diffusivity
ψ	stream function
γ	constant
θ	dimensionless temperature
δ	boundary layer thickness

divided into two parts by a plate with the fluid temperature in the left side being T_f and the fluid temperature in the right side being T_∞. The plate is heated or cooled by convection of the fluid in the left side. The Cartesian coordinate system (x, y) is chosen with the x-axis being measured along the plate and the y-axis being normal to it. With the assumptions that (1) the fluid and the porous medium are everywhere in local thermodynamic equilibrium, (2) the temperatures for fluids in both sides of the plate are below boiling point, (3) properties of the fluid and the porous medium are constant, (4) the fluid and the porous medium are isotropic, and (5) the Boussinesq approximation is applied, the governing equations for this problem can then be written as (see Cheng and Minkowycz [1])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u=-\frac{K}{\mu}\left(\frac{\partial p}{\partial x}+\rho g\right), \tag{2}$$

$$v=-\frac{K}{\mu}\frac{\partial p}{\partial y}, \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

$$\rho=\rho_\infty[1-\beta(T-T_\infty)], \tag{5}$$

subject to the following boundary conditions as suggested by Aziz [9]

$$v=0\,,\quad -k_m\frac{\partial T}{\partial y}(x,0)=h_f(x)\left[T_f-T(x,0)\right]\quad at\quad y=0, \tag{6}$$

$$u\to 0\,,\quad T\to T_\infty\quad as\quad y\to\infty, \tag{7}$$

where u and v are Darcy's velocity in the x and y directions respectively, p is the pressure, T is the temperature, μ, ρ, and β are, respectively, the dynamic viscosity, the density and the thermal expansion coefficient of the fluid, K is the permeability of the saturated porous medium, g is the acceleration due to gravity, ρ_∞ is the density of the ambient fluid, $\alpha = k_m/(\rho_\infty C_p)_f$ is the equivalent thermal diffusivity with k_m being the thermal conductivity of the saturated porous medium and C_p being the specific heat of the fluid, and $h_f(x)$ is the heat transfer coefficient due to T_f.

We introduce the following similarity transformations

$$\psi=\alpha {Ra_x}^{1/2}f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\quad \eta=\frac{y}{x}{Ra_x}^{1/2}, \tag{8}$$

where $\psi(x, y)$ is the stream function defined in a usual form as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, $Ra_x = \rho_\infty K\beta g(T_f - T_\infty)x/(\alpha\mu)$ is the modified local Rayleigh number in a porous medium.

Based on the above similarity variables, the velocity components for u and v are expressed by

$$u=\frac{\partial\psi}{\partial y}=\left[\rho_\infty g\beta\left(T_f-T_\infty\right)K/\mu\right]f'(\eta), \tag{9}$$

$$v=\frac{1}{2}\left[\alpha\rho_\infty g\beta K\left(T_f-T_\infty\right)/(\mu x)\right]^{1/2}(\eta f'-f), \tag{10}$$

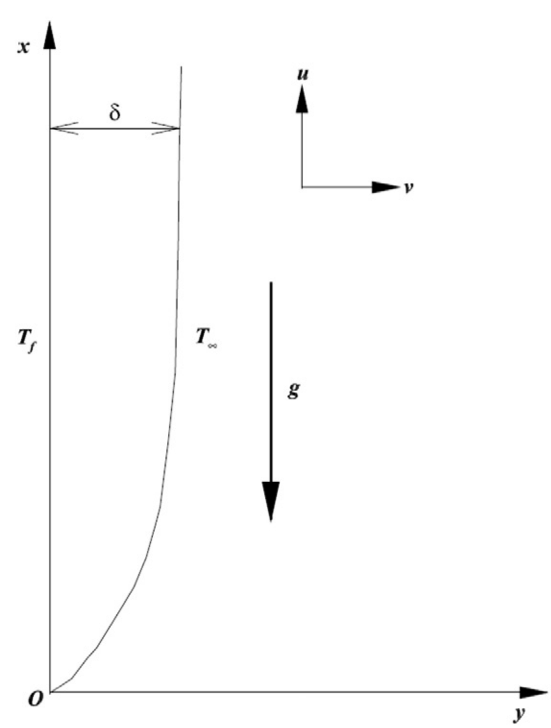

Fig. 1. Physical configuration and coordinate system.