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Analysis of heatlines and entropy generation during free convection within trapezoidal cavities ☆

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ABSTRACT

In this article natural convection flows have been studied within trapezoidal cavities where left wall of the cavity is hot and right wall is maintained at constant cold temperature while the top and bottom walls are adiabatic. The results are presented in terms of streamlines, heatlines, isotherms, entropy generation due to fluid friction, entropy generation due to heat transfer, average Bejan number, total entropy generation and average Nusselt number. It may be concluded that, the trapezoidal cavity with $\varphi = 60°$ is the optimal shape for thermal processing at $Pr = 0.015$ whereas square cavity ($\varphi = 90°$) is the optimal design for the thermal processing at $Pr = 7.2$ based on lower S_{total} and higher $\overline{Nu_l}$.

1. Introduction

Free convection phenomenon has been studied extensively due to its various engineering applications, such as electronic cooling [1], geothermal [2], solar collector [3], thermal energy storage [4] etc. Several investigations have been conducted on free convection within square and trapezoidal cavities [5–8]. However, most of the above studies are based on streamlines and isotherms and the detailed analysis of heat flow is not well understood. Kimura and Bejan [9] proposed heatline concept to analyze heat flow patterns in two dimensional convective heat transport process. Recently, the heatline concept is used to understand heat flow patterns within square cavity with multiple discrete heat source-sink pairs [10], horizontal planar square cavity with discrete heat sources flush-mounted on its bottom wall [11], two dimensional square cavity with wavy right wall [12].

Eventhough, the above investigations are carried out to understand the flow and isotherm patterns within enclosures, these studies are unable to explain the thermal efficiency of the system. In order to improve the system thermodynamically, a new methodology called as exergy analysis and its optimization tool entropy generation minimization is introduced. Entropy generation minimization results in minimum irreversibilities associated with the process and thus the overall efficiency of the system is increased. Therefore by analyzing the entropy generation due to heat transfer and fluid flow irreversibilities, the strategies to optimize the process may be achieved to increase the overall efficiency of the system. Bejan [13–15] introduced entropy generation minimization concept based on the second law of thermodynamics. Significant amount of work has been done on entropy generation minimization for various applications [16–18]. Famouri and Hooman [16] investigated entropy generation for natural convection in a partitioned cavity where vertical walls are isothermally cooled and horizontal walls are adiabatic. Mukhopadhyay [17] studied entropy generation due to natural convection in a square enclosure heated locally from below with two isoflux heat sources. Ilis et al. [18] analyzed entropy generation in rectangular cavities with the same area but different aspect ratios. Till date, analysis on entropy generation during natural convection within trapezoidal cavities in the presence of hot and cold side walls with adiabatic horizontal walls is yet to appear in literature.

The objective of the present investigation is to analyze the heat flow visualization on heatline approach and entropy generation during natural convection within trapezoidal cavities whereas the left wall is hot and right wall is maintained at constant cold temperature while the top and bottom walls are adiabatic. In the current study, the Galerkin finite element method has been employed to solve the nonlinear equations of fluid flow, energy and entropy.

2. Mathematical modeling and simulation

Let us consider a trapezoidal cavity with the right wall inclined at an angle $\varphi = 30°$, $60°$ and $90°$ with the X-axis as seen in Fig. 1a–c, respectively. The boundary conditions for velocities are considered as no-slip on solid boundaries. The fluid is considered as incompressible, Newtonian and the flow is assumed to be laminar. For the treatment of the buoyancy term in the momentum equation, Boussinesq approximation is employed for the equation of the vertical component of velocity to account for the variations of density as a function of

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Nomenclature

Be	Bejan number
g	Acceleration due to gravity, m s^{-2}
L	Length of the square cavity, m
Nu	Local Nusselt number
$\overline{Nu}$	Average Nusselt number
p	Pressure, Pa
P	Dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
S_ψ	Dimensionless entropy generation due to fluid friction
S_θ	Dimensionless entropy generation due to heat transfer
S_{total}	Dimensionless entropy generation due to fluid friction and heat transfer
T	Temperature of the fluid, K
T_h	Temperature of hot left wall, K
T_c	Temperature of cold right wall, K
u	x component of velocity, m s^{-1}
U	x component of dimensionless velocity
v	y component of velocity, m s^{-1}
V	y component of dimensionless velocity
X	Dimensionless distance along x coordinate
Y	Dimensionless distance along y coordinate

Greek symbols

α	Thermal diffusivity (m^2 s^{-1})
β	Volume expansion coefficient (K^{-1})
γ	Penalty parameter
θ	Dimensionless temperature
ν	Kinematic viscosity (m^2 s^{-1})
ρ	Density (kg m^{-3})
φ	Inclination angle with the positive direction of X axis
ψ	Dimensionless streamfunction

temperature and to couple in this way the temperature field to the flow field. The governing equations for steady natural convection flow using conservation of mass, momentum and energy in dimensionless form can be written as:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0\ , \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\,Pr\,\theta, \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\ . \tag{4}$$

where

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{uL}{\alpha},V=\frac{vL}{\alpha},\theta=\frac{T-T_c}{T_h-T_c},$$

$$P=\frac{pL^2}{\rho\alpha^2},\quad Pr=\frac{\nu}{\alpha},\quad Ra=\frac{g\beta(T_h-T_c)L^3}{\nu\alpha}, \tag{5}$$

with following boundary conditions

$$\begin{gathered}U=0,V=0,\frac{\partial\theta}{\partial Y}=0,\forall Y=0,0\le X\le 1\\ U=0,V=0,\theta=1,\forall Xsin(\varphi)+Ycos(\varphi)=0,0\le Y\le 1\\ U=0,V=0,\theta=0,\forall Xsin(\varphi)-Ycos(\varphi)=sin(\varphi),0\le Y\le 1\\ U=0,V=0,\frac{\partial\theta}{\partial Y}=0,\forall Y=1,-cot(\varphi)\le X\le 1+cot(\varphi)\end{gathered} \tag{6}$$

The momentum and energy balance equations (Eqs. (2)–(4)) are solved using the Galerkin finite element method. The continuity equation (Eq. (1)) is used as a constraint due to mass conservation and this constraint may be used to obtain the pressure distribution. In order to solve Eqs. (2) and (3), we use the penalty finite element method where the pressure, P, is eliminated by a penalty parameter γ and the incompressibility criteria given by Eq. (1) as results in

$$P=-\gamma\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)\ . \tag{7}$$

The continuity equation (Eq. (1)) is automatically satisfied for large values of γ. Typical values of γ that yield consistent solutions are 10^7. Using Eq. (7), the momentum balance equations (Eqs. (2) and (3)) reduce to

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\gamma\frac{\partial}{\partial X}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+Pr\ \left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{8}$$

and

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=\gamma\frac{\partial}{\partial Y}\left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}\right)+Pr\ \left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\ \ Pr\ \ \theta\ . \tag{9}$$

The system of equations (Eqs. (4), (8) and (9)) with appropriate boundary conditions (Eq. (6)) are solved using Galerkin finite element method [19]. Since the solution procedure is explained in an earlier work [20], the detailed description is not included in this paper. The numerical solutions are obtained in terms of the velocity components (U,V).

2.1. Streamfunction, Nusselt number, heatfunction and entropy generation

The streamfunction (ψ) is evaluated using the relationship between the streamfunction (ψ) and the velocity components, where the streamfunction (ψ) is defined as

$$U=\frac{\partial\psi}{\partial Y}\quad \text{and}\quad V=-\frac{\partial\psi}{\partial X}\ . \tag{10}$$

Positive sign of ψ denotes anti-clockwise circulation and clockwise circulation is represented by negative sign of ψ. The no-slip condition ($\psi=0$) is valid at all boundaries as there is no cross flow. The heat transfer coefficient in terms of the local Nusselt number (Nu) is defined by

$$Nu=-\frac{\partial\theta}{\partial n}\ , \tag{11}$$

where n denotes the normal direction on a plane. The local Nusselt numbers along left wall (Nu_l) and right wall (Nu_r) are defined as

$$Nu_l=\left(sin\varphi\frac{\partial\theta}{\partial X}+cos\varphi\frac{\partial\theta}{\partial Y}\right)\quad\text{and}\quad Nu_r=-\left(sin\varphi\frac{\partial\theta}{\partial X}-cos\varphi\frac{\partial\theta}{\partial Y}\right)\ . \tag{12}$$