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Finite element simulations on heat flow visualization and entropy generation during natural convection in inclined square cavities☆

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ABSTRACT

Finite element based numerical simulation has been carried out for analysis of heat flow visualization and entropy generation during natural convection within inclined square cavities with hot wall (DA), cold wall (BC) and adiabatic walls (AB and CD). The numerical results are presented in terms of isotherms (θ), streamlines (ψ), heatlines (Π), entropy generation due to heat transfer irreversibility (S_θ) and fluid friction irreversibility (S_ψ). Further, detailed discussion on variation of the total entropy generation (S_{total}), average Bejan number (Be_{av}) and average Nusselt number ($\overline{Nu}$), with Rayleigh number (Ra) is also presented. It is found that, large heat transfer rate ($\overline{Nu}_{DA}$) with less entropy generation (S_{total}) occurs for $\varphi = 15°$ cavities at convection dominant mode ($Ra = 10^5$) irrespective of Pr. Thus, inclined square cavities with $\varphi = 15°$ may be used for all thermal processing operations involving various fluids ($Pr = 0.025$ and 998).

1. Introduction

The heat transport in buoyancy driven flow is a mechanism termed as natural convection which results due to the density difference in the fluid, induced by temperature gradient. Natural convection phenomena in inclined cavities has received large attention in the recent years due to various applications as design of solar collectors [1,2], thermal performs of plate-fin heat sink [3] and cooling of electronic components [4].

The studies on natural convection within inclined cavities are analyzed by few investigators [5–9]. In the previous studies on natural convection, the fluid flow visualization via streamlines and temperature distribution are given much importance while the thermal management in heat flow is poorly understood. Thus, in order to complete understanding of heat distribution, one has to study the heatlines which indicate the trajectory of heat flow. The basic concept of heatline was explained by Kimura and Bejan [10]. Further, the heatline concept was studied by few investigators Zhao et al. [11] and Dalal and Das [12].

Although study of heatlines provides us the information about the heat distribution inside the cavity, it is not adequate to explain about the efficiency of heating processes. It does not account for the loss in the available energy which is required to heat the fluid. Thus, to analyze the efficiency of the heating process one has to analyze the entropy generation within the system because the loss in available energy is directly proportional to the entropy generation inside the cavity. Entropy can be defined as the randomness of any thermodynamic process. Entropy generation brings down the overall energy efficiency of the process and increases the wastage in useful form of energy. Bejan [13,14] introduced entropy generation minimization concept based on the second law of thermodynamics. Entropy generation inside a square cavity during laminar natural convection was studied by Erbay et al. [15]. A detailed investigation on entropy generation due to heat transfer and fluid friction irreversibilities during natural convection within various cavities was done by Alipanah et al. [16], Kuddusi [17] and Bouabid et al. [18]. However, the analysis of entropy generation with correlation on heat distribution via heatline concept during natural convection within inclined cavities with Rayleigh–Benard convection is yet to appear in the literature.

The main objective of the present study is to analyze the effect of inclination angle on the fluid flow via streamline, heat flow via heatlines and entropy generation due to heat transfer and fluid friction during natural convection within an inclined square enclosure. The enclosure is bounded by two adiabatic walls (AB and CD), hot wall (DA) and cold wall (BC) [see Fig. 1]. Numerical simulations were carried out for various parameters as Rayleigh numbers ($10^3 \leq Ra \leq 10^5$), Prandtl numbers ($Pr = 0.025$ and 998) and inclination angles as $\varphi = 15°$, 30° and 60°. The Galerkin finite element method [19] with a penalty parameter is used in order to solve the non-linear partial differential equations.

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Nomenclature

Be	Bejan number
g	acceleration due to gravity, m s^{-2}
L	length of the tilted square cavity, m
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
S	dimensionless entropy generation
S_ψ	dimensionless entropy generation due to fluid friction
S_θ	dimensionless entropy generation due to heat transfer
S_{total}	dimensionless entropy generation due to fluid friction and heat transfer
T	temperature of the fluid, K
T_h	temperature of hot left wall, K
T_c	temperature of cold right wall, K
u	x component of velocity, m s^{-1}
U	x component of dimensionless velocity
v	y component of velocity, m s^{-1}
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
Y	dimensionless distance along y coordinate

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	volume expansion coefficient (K^{-1})
θ	dimensionless temperature
ν	kinematic viscosity (m^2 s^{-1})
ρ	density (kg m^{-3})
φ	inclination angle with the positive direction of X axis
ψ	dimensionless streamfunction
Π	dimensionless heatfunction

2. Mathematical formulation, simulation and post processing

2.1. Velocity and temperature distributions

The governing equations for the steady natural convective flow for the given problem by using the conservation of mass, momentum, and energy in dimensionless form are given as below:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0, \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\ Pr\ \theta, \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}, \tag{4}$$

where

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{uL}{\alpha},\quad V=\frac{vL}{\alpha},\quad \theta=\frac{T-T_c}{T_h-T_c}$$
$$P=\frac{pL^2}{\rho\alpha^2},\quad Pr=\frac{\nu}{\alpha},\quad Ra=\frac{g\beta(T_h-T_c)L^3Pr}{\nu^2}. \tag{5}$$

The boundary conditions for temperature and velocity profiles are given as,

$$\begin{aligned}&U(X,Y)=0,\ \ V(X,Y)=0\ \ n.\nabla\theta=0\ \ \text{along walls AB and CD}\\&U(X,Y)=0,\ \ V(X,Y)=0\ \ \theta=0\ \ \text{along wall BC},\\&U(X,Y)=0,\ \ V(X,Y)=0\ \ \theta=1\ \ \text{along wall DA}.\end{aligned} \tag{6}$$

The continuity equation [Eq. (1)] is used as a constraint due to mass conservation and this constraint can be used to obtain the pressure distribution by penalty formulation. The momentum and energy balance equations [Eqs. (2)–(3)] are solved using Galerkin finite element method [19]. Since the solution procedure is explained in an earlier work [20], the detailed description is not included in this paper.

2.2. Streamfunction, Nusselt number, heatfunction and entropy generation

The pattern of the fluid flow inside the enclosure is represented via using the streamfunction (ψ) that is given from the velocity components (U and V) as:

$$\frac{\partial^2\psi}{\partial X^2}+\frac{\partial^2\psi}{\partial Y^2}=\frac{\partial U}{\partial Y}-\frac{\partial V}{\partial X}, \tag{7}$$

where the positive sign of ψ denotes anticlockwise circulation and clockwise circulation is represented by negative sign of ψ. The no-slip condition is valid at all boundaries as there is no cross flow. The heat transfer coefficient in terms of local Nusselt number (Nu) is defined as the following:

$$Nu=-\frac{\partial\theta}{\partial n}, \tag{8}$$

where n denotes the unit normal direction on a plane. The local Nusselt numbers at walls BC (Nu_{BC}) and DA (Nu_{DA}) are given as:

$$Nu_{BC}=-\left(cos\varphi\frac{\partial\theta}{\partial X}+sin\varphi\frac{\partial\theta}{\partial Y}\right)\quad\text{and}\quad Nu_{DA}=\left(cos\varphi\frac{\partial\theta}{\partial X}+sin\varphi\frac{\partial\theta}{\partial Y}\right). \tag{9}$$

The average Nusselt numbers at either of the adiabatic walls (BC or DA) are given as:

$$\overline{Nu_s}=\int_0^1 Nu_s dS, \tag{10}$$

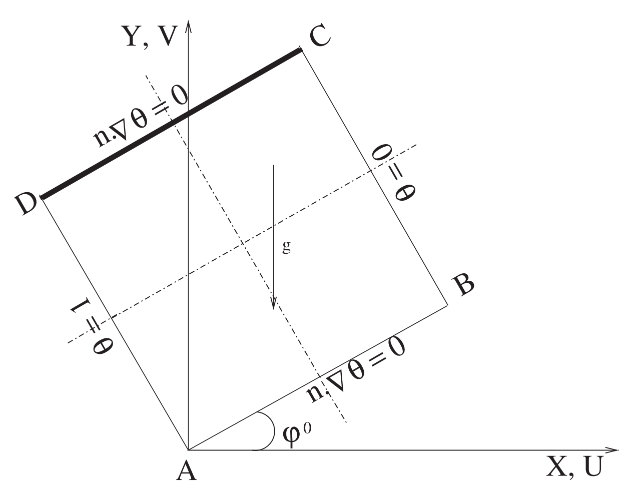

Fig. 1. Schematic diagram of the computation domain with the boundary conditions. The dashed line represents the line of geometric symmetry. The inclination angles of the tilted cavity have been taken as $\varphi = 15°$, 30° and 60°.