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An evaporation model for oscillating spheroidal drops ☆

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ABSTRACT

The evaporation process of a liquid spheroidal drop floating in a gaseous atmosphere has been modelled, accounting for the oscillation between oblate and prolate states. A previously developed exact solution for the heat and mass transfer equations has been extended to investigate the effect of oscillation on drop evaporation under the assumption of quasi steady-state conditions and the results are compared with approximate models from the open literature. The validity of the quasi steady-state assumption is discussed, deriving, for different fluids, the range of drop temperature and size and gas temperature where it is reasonably acceptable.

1. Introduction

The growing practical interest in spray evaporation and combustion has motivated the request of a detailed understanding of the different phenomena taking place when a liquid droplet floats in a gaseous atmosphere, with an associated increasing demand for analytical/empirical correlations describing the interphase transfer of mass, momentum and energy [1].

The process of liquid drop vaporization has attracted the researchers since the nineteenth century. The simplest model for the droplet evaporation rate was proposed by Maxwell back in 1877 [2], which suggested that the driving force for liquid evaporation is the difference in vapour concentration between the drop surface and the free stream and the process is exclusively controlled by the diffusion mechanism.

Since then, a variety of different models have been proposed in order to capture the complexity of the physical phenomena involved in the process, including the bulk motion of the gas surrounding the droplet (Stefan flow) [3], the heat and mass diffusion in the droplet interior [4], the liquid composition [5,6], gas stream effect [7] and high-pressure effect [8,9]. Refer to [10] for a recent review of the main developments in modelling droplet heating and evaporation.

One assumption that yet prevails in most of the theoretical/empirical models widely used in commercial CFD codes for dispersed phase (like sprays, or particle laden flows) application is that liquid droplets maintain spherical shape while interacting with the gaseous phase [7]. However, significant shape deformations are expected and observed while liquid drops interact with the carrier phase, and these deformations are of fundamental importance for understanding many natural and industrial processes involving spray droplets [11,12].

Non-spherical liquid drops are unstable and the opposing effects of surface tension and inertia cause periodic or non-periodic variation of the drop shape, which is referred to as drop oscillation and it is found to strongly influence heat, momentum and mass transfer between liquid drops and the surrounding gas [13]. Oscillation can become important in atomisation systems where the liquid is first disintegrated into small ligaments, which then oscillate towards the asymptotic attainment of an equilibrium spherical shape [14].

One of the pioneering works on non-spherical droplet dynamics was that of Lamb [15], which led to an expression for the natural frequency of infinitesimal amplitude oscillations of an inviscid drop immersed in an inviscid quiescent environment.

Afterwards, a considerable amount of work has been done on the dynamics of oscillating drops; refer to [16–22] for reviews on numerical and experimental contributions to this field. Effect of viscosity was considered by many researchers, among other results it was shown that viscosity effects are responsible of a relatively quick damping of the highest oscillation modes, then living only the oblate–prolate mode ($n = 2$) to survive [23].

All these works address the issue of oscillating drop/ligament dynamics under non-evaporating conditions. When the drops are exposed to hot gas, the heat transfer and the consequent evaporation could affect and be affected by oscillations [24–26].

Despite the large amount of work done over the last decades on non-evaporating liquid drop oscillation, to the best of authors' knowledge only few papers can be found in the open scientific literature addressing the effect of evaporation on oscillating drops [27–29], and the few available experimental data-sets do not report all the necessary information for model comparison.

Deng et al. [30] proposed one of the early numerical studies on this subject with a two-dimensional numerical model investigating the dynamics of non-evaporating and evaporating liquid ligaments undergoing deformation/breakup and oscillations under viscous convective flows. The results showed that the dynamics of ligament deformation was basically unaffected by vaporization, however the evaporation rates (per unit area) are greater for deformed drops.

Mashayek [31] suggested a correlation for the rate of evaporation of deformed drop based on the results from numerical simulations, which

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Nomenclature

Greek symbols

α	thermal diffusivity	m^2/s
β	surface area ratio	–
γ	specie flux	kg/m^2s
ϵ_n	non-dimensional disturbance amplitude	–
ε	deformation parameter $\varepsilon = \frac{a_z}{a_r}$	–
ζ, θ, φ	spheroid coordinates	–
ρ	density	kg/m^3
σ	surface tension	kg/s^2
τ	oscillation period	s
χ	mass fraction	–
ω	oscillation frequency	$1/s$
Γ	evaporation enhancement	–
Δ	relative difference	–
Θ	non-dimensional evaporation rate parameter	–
Ξ	mean curvature	–
Π	non-dimensional vapour flux	–

Roman symbols

a	perturbation parameter	–
a_r, a_z	spheroid axes	m
C_p	specific heat	J/kgK
D_v	diffusivity	m^2/s
k	thermal conductivity	W/mK
$\dot{m}_{ev}$	evaporation rate	kg/s
n	oscillation mode	–
P_n	Legendre polynomials	–
R	drop radius	m
t	time	s
T	temperature	K
U	velocity	m/s
x, y, z	Cartesian coordinates	–

Subscripts

α	species index	-
$conv$	convective	
$diff$	diffusive	
cd	convective–diffusive	
$evap$	evaporation	
g	gaseous	
l	liquid	
$oscil$	oscillation	
$0, s$	surface	
v	vapour	
∞	infinity	

Superscripts

~	non-dimensional
H	heat transfer

showed that the mass flux varies along the surface of the deformed drop. The Author proposed a correlation to express the mass flux as a function of the surface curvature, based on the suggestion of Lian and Reitz [32] who studied the instability of evaporating liquid jets, postulating that the deformed surface may be locally considered as the surface of a spherical drop having the same mean curvature as that of the deformed surface, and that the local flux would be that of a spherical drop with that curvature radius.

Recently [33], it has been shown that an analytical solution of the steady state heat and mass transfer equations exists for spheroidal (oblate and prolate) drops floating in a gaseous atmosphere, and the local evaporation rate was exactly correlated with the local surface curvature.

The evaporation from free oscillating particles was investigated in [34], showing that the increase in the evaporation rate of an oscillating drop is proportional to the square of the surface disturbance amplitude and is larger for higher oscillation modes, and that the period of oscillation is decreased by evaporation, while the dominant mode of oscillation remains the same as that for a non-evaporating drop.

The present work was motivated by the necessity to include the above described complex drop evaporation mechanisms in spray numerical simulations, using relatively simple sub-models for predicting the inter-phase phenomena taking place during the spray evolution. The following sections report the mathematical model, the derivation of analytical expressions for the instantaneous and average evaporation rate and heat rate from oscillating spheroidal liquid drops, the comparison against the predictions of previously available models and the derivation of conditions for model applicability for different fluids. The main conclusions are then briefly summarised.

2. The instantaneous vapour flux and heat rate for spheroidal drops

For a liquid drop made of a single component floating in a gaseous atmosphere, the species conservation equations can be written [35]:

$$\rho U_j \nabla_j \chi_\alpha = \nabla_j \left(\rho D_v \nabla_j \chi_\alpha \right) \tag{1}$$

where $\alpha = v$, g refers to the vapour and gaseous phases respectively, while $\chi_\alpha = \frac{\rho_\alpha}{\rho}$ is the mass fraction and D_v is the binary diffusion coefficient; the symmetry of the diffusion coefficients $D_v = D_{vg} = D_{gv}$ for a binary mixture is imposed according to [35].

In the following, gas density is assumed to be constant, according to the majority of evaporation models for spray simulations [36]. Accounting for density gradient effect on spherical droplet evaporation [37] led to the conclusion that the constant gas density assumption may become questionable for very high gas temperature evaporating conditions.

Setting to constant values the vapour mass fraction at drop surface ($\chi_v = \chi_{v,s}$) and at infinite distance from the drop ($\chi_v = \chi_{v,\infty}$), an analytical solution of (1) was proposed in [33], through the use of prolate and oblate spheroidal coordinate systems, defined as:

$$\begin{aligned} x &= a\ A(\xi) sin(\theta) cos(\varphi) \\ y &= a\ A(\xi) sin(\theta) sin(\varphi) \\ z &= a\ B(\xi) cos(\theta) \end{aligned}$$

where:

$$\begin{aligned} A(\xi) &= \cosh(\xi); B(\xi) = \sinh(\xi); \quad \text{for oblate case} \\ A(\xi) &= \sinh(\xi); B(\xi) = \cosh(\xi); \quad \text{for prolate case} \end{aligned}$$

In these coordinate systems, the spheroid surface equation is $\xi = \xi_0$, and the above-mentioned B.C. are:

$$\chi_v(\xi_0, \theta, \varphi) = \chi_{v,s}; \quad \chi_v(\infty, \theta, \varphi) = \chi_{v,\infty} .$$

The steady state analytical solution of the balance Eq. (1) provides the following form for the local instantaneous vapour flux:

$$\gamma_{v,s} = \frac{\rho D_v}{R_0} \frac{\varepsilon^{2/3}}{\left[1 - \left(1 - \varepsilon^2 \right) sin^2(\theta) \right]^{1/2}} \Gamma(\varepsilon) ln \frac{1 - \chi_{v,\infty}}{1 - \chi_{v,s}} \tag{2}$$

where the deformation parameter ε is defined as:

$$\varepsilon = \frac{a_z}{a_r}$$