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Effect of magnetic obstacle on fluid flow and heat transfer in a rectangular duct☆

Xidong Zhang, Hulin Huang *

Academy of Frontier Science, Nanjing University of Aeronautics and Astronautics, No. 29 Yudao Street, Nanjing, Jiangsu 210016, PR China

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ABSTRACT

The vortex dynamics behind various magnetic obstacles and characteristics of heat transfer are investigated using a three-dimensional model. In the numerical study, the magnet width (M_y) is alterable to investigate the instability, Strouhal number, wake structure behind various magnetic obstacles and percentage increment of the overall heat transfer for a wide range of constrainment factors ($0.08 \le \kappa \le 0.26$), Reynolds numbers ($400 \le Re \le 900$) and interaction parameters ($9 \le N \le 15$). For all constrainment factors, the fundamental frequency (f) is uniform for a particular value of Reynolds number. Downstream cross-stream mixing due to vortex shedding enhances the wall-heat transfer and the maximum value of percentage increment of the overall heat transfer (HI) is about 20.2%. However, the pressure drop penalty ($\Delta P_{\text{penalty}}$) is not increasingly dependent on interaction parameter when Re and κ remain constant.

1. Introduction

Convection heat transfer has an important role in myriad practical applications. Examples of forced convection heat transfer and natural convection heat transfer are very much common in engineering applications such as tokamak confinement fusion devices, space heating, cooling towers, power generators, heat losses from high-rise buildings, heat exchanger, solar collector and other thermal applications [1–5].

The motion of an electrically conducting fluid under an external magnetic field can induce electric currents, which in turn interact with the magnetic field resulting in a Lorentz force. This has a significant effect on the velocity distribution and the turbulence characteristics. For example, a liquid metal passing a localized magnetic field can exhibit some features similar to those observed in ordinary hydrodynamics of fluid flow around a circular cylinder. These hydrodynamic peculiarities and prediction of heat transfer for fluid flow around a cylinder, because of their practical importance in hydrodynamic and heat transfer applications, have been intensely studied [6–9]. However, the flow around a magnetic obstacle is a rather new magnetohydrodynamic (MHD) problem that is not yet qualitatively well understood. Moreover, the importance of understanding the flow around a magnetic obstacle is evident because any real magnetic field will be always nonuniform. Cuevas et al. [10,11] analyzed numerically two-dimensional and quasi-two-dimensional flow past a local magnetic field at low Reynolds numbers 100 and 200. However, their results are justified for creeping systems only and 2D results are questionable for the real system when Re is high. Votyakov et al. [12,13] thought that the two-dimensional numerical simulation did not well explore the complex flow structures of the magnetic obstacle at high Reynolds number. Therefore they derived, for the first time, the equations of the external magnetic field and new stationary MHD flow patterns, and compared the results of 3D numerical simulations with physical experiments. Votyakov and Kassinos [14] reported new recirculation patterns and discuss a fundamental difference in 2D and 3D systems with magnetic obstacle, and explain why 2D simulation reveals multi-vortex effects when N is very large. Andreev et al. [15] devised an experiment of the liquid metal past a magnetic obstacle in a rectangular channel and proved that the interaction parameter N governed the flow when turbulent pulsations were suppressed by the external magnetic field. Votyakov and Kassinos [16] reported the unsteady flow past a magnetic obstacle. They showed the breaking away of attached vortices from the magnetic obstacle when the Reynolds number was large enough. Zhang and Huang [17] investigated the effect of blockage ratio on the fluid flow and heat transfer at constant magnet width. The results show that the value of Strouhal number increases as the blockage ratio (β) increases, and for small β the variation of St is very small when the interaction parameter and Reynolds number are increasing. The maximum of percentage heat transfer increment is about 50.5% at $\beta = 0.4$.

A review of the literatures finds just 10 papers concerning the heat transfer and vortex shedding characteristics of an electrically conducting fluid past a magnetic obstacle, compared with the thousands of papers published for ordinary solid obstacles. Therefore, the aim of the present work is to study the dynamics and heat transfer characteristics in MHD channel flow past different magnetic obstacles. In particular, the effect of constrainment factor ($\kappa = M_y / L_y, L_y = 0.2$) on the structure of the flow and heat transfer will be investigated.

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* Corresponding author.
E-mail address: hlhuang@nuaa.edu.cn (H. Huang).

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Nomenclature

$\boldsymbol{B}$	magnetic field vector (T)
$\boldsymbol{B}_0$	applied field vector (T)
$\boldsymbol{b}$	induced magnetic field vector (T)
$\boldsymbol{F}$	Lorentz force (N/m^3)
$\boldsymbol{J}$	induced current (A/m^2)
P	pressure (Pa)
t	time (s)
T	temperature field (K)
T_∞	free stream temperature (K)
T_w	temperature of hot channel side-wall (K)
$\boldsymbol{u}$	velocity vector (m/s)
M_x, M_y, H	characteristic magnet dimensions (m)
U	the area-averaged inflow velocity (m/s)
L	characteristic dimension $L = L_z/2$ (m)
f	vortex shedding frequency (1/s)
τ_p	period of vortex shedding (s)
Nu	local Nusselt number
$\langle Nu \rangle$	surface-averaged Nusselt number
$\langle \overline{Nu} \rangle$	time and surface-averaged Nusselt number
Ha	Hartmann number
St	Strouhal number, $St = f\frac{M_y}{U}$
Re	Reynolds number
N	interaction parameter
Re_m	magnetic Reynolds number
μ_m	magnetic permeability (H/m)
HI	percentage increment of heat transfer
Pr	Prandtl number

Greek symbols

α	thermal diffusivity (m^2/s)
K	constrainment factor define by M_y/L_y
ν	kinematic viscosity (m^2/s)
ρ	fluid density (kg/m^3)
σ	electrical conductivity (1/Ω·m)

Subscripts

w	wall
m	magnetic
0	absent external magnetic field

2. Numerical modeling

2.1. Setup

The numerical setup is a rectangular duct as shown in Fig. 1 with a length ($L_u + L_d$) of 0.75 m, width (L_y) of 0.2 m, and height (L_z) of 0.02 m, respectively. The duct is filled with the eutectic alloy GaInSn with a typical composition of 68.5% Ga, 21.5% In, and 10% Sn. A Prandtl number $Pr = 0.020$ is used throughout. The characteristic length scale is taken to be the half-channel height ($L = L_z/2$) and the characteristic velocity to be the averaged fully developed Poiseuille velocity profile. Throughout this study, the channel width L_y is fixed and the magnetic constrainment factor which defines the spanwise distribution of the magnetic field with $\kappa = M_y / L_y = 0.08$, 014, 0.2 and 0.26 is chosen in this investigation.

2.2. Governing equations and boundary conditions

The energy/mass balance equations and partial differential equations derived from the Navier–Stokes equation coupled with the Maxwell equations for a moving medium and Ohm's law are solved numerically. In our model it is assumed that the flow is laminar, incompressible and Newtonian with constant properties. The effect of Joule heating is neglected. Based on these assumptions, the governing equations in non-dimensional form become

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial \tau} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{V} = -\nabla P^* + \frac{1}{Re}\nabla^2 \boldsymbol{V} + \boldsymbol{N}(\boldsymbol{J}^* \times \boldsymbol{B}^*) \tag{2}$$

$$\frac{\partial \Theta}{\partial \tau} + (\boldsymbol{V} \cdot \nabla)\Theta = \frac{1}{Pe}\nabla^2 \Theta. \tag{3}$$

The induced magnetic field equation under the classical MHD assumptions can be derived [19]:

$$\frac{\partial \boldsymbol{b}^*}{\partial \tau} + (\boldsymbol{V} \cdot \nabla)\boldsymbol{B}^* = \frac{1}{Re_m}\nabla^2 \boldsymbol{b}^* + (\boldsymbol{B}^* \cdot \nabla)\boldsymbol{V} \tag{4}$$

where $\boldsymbol{B}^*$ is the magnetic field intensity ($\boldsymbol{B}^* = \boldsymbol{B}_0^* + \boldsymbol{b}^*$). The applied magnetic field $\boldsymbol{B}_0^*$ satisfies the magnetostatic equations [10,13], namely,

$$\nabla \cdot \boldsymbol{B}_0^* = 0 \quad \text{and} \quad \nabla \times \boldsymbol{B}_0^* = 0. \tag{5}$$

So the induced field implicitly satisfies the equation

$$\nabla \cdot \boldsymbol{b}^* = 0. \tag{6}$$

Once Eq. (4) is solved for the induced magnetic field, the induced current can be deduced from the Ampere's law, $\boldsymbol{J}^* = \frac{1}{Re_m}(\nabla \times \boldsymbol{B}^*)$. The following dimensionless variables and parameters have been used:

$$\boldsymbol{V} = \frac{\boldsymbol{u}}{U}, \quad \tau = \frac{tU}{L}, \quad \Theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad P^* = \frac{P}{\rho U_\infty^2}, \quad \boldsymbol{J}^* = \frac{\boldsymbol{J}}{B_0 \sigma U_\infty}, \quad \boldsymbol{B}^* = \frac{\boldsymbol{B}}{B_0},$$

$$Re = \frac{UL}{\nu}, \quad Pe = RePr,$$

$$(X, Y, Z) = \frac{(x, y, z)}{L}, \quad \mathrm{Ha} = B_0 L\sqrt{\frac{\sigma}{\rho \nu}}, \quad N = \frac{Ha^2}{Re}, \quad Re_m = \mu_m \sigma UL.$$

The variables have their usual sense in fluid mechanics and heat transfer as listed in the nomenclature.

The external magnetic field with a strength B_0 can be obtained from a semianalytical simplification of the Biot–Savart's and Maxwell equations, Votyakov et al. [13,18]:

$$B_\alpha(x, y, z) = \gamma \sum_{k=\pm1} \sum_{j=\pm1} \sum_{i=\pm1} (ijk) A_\alpha \tag{7}$$

where $\alpha = x, y, z$ are the magnetic field components, and γ is a normalization constant. The external magnetic field B_0 is obtained in such a way that B_{0z} (0, 0, 0) $= B_0$ with functions:

$$A_x = \mathrm{artanh}\left[\frac{y - 0.5jM_y}{r(i, j, k)}\right], \quad A_y = \mathrm{ar\,tanh}\left[\frac{x - 0.5iM_x}{r(i, j, k)}\right],$$

$$A_z = -\mathrm{arc\,tan}\left[\frac{(z - 0.5kH)r(i, j, k)}{(x - 0.5iM_x)\left(y - 0.5jM_y\right)}\right], \tag{8}$$

$$r(i, j, k) = \left[(x - 0.5iM_x)^2 + \left(y - 0.5jM_y\right)^2 + (z - 0.5kH)^2\right]^{1/2}.$$