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High order numerical simulation of non-Fourier heat conduction: An application of numerical Laplace transform inversion ☆

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ABSTRACT

Non-Fourier heat conduction phenomenon in a finite slab with insulated boundaries is investigated in the present paper. Since solving the hyperbolic heat conduction equation analytically requires considerable effort, a new high-order numerical approach has been implemented to achieve comparable exactitude. This method solves the considered equation in Laplace space and numerical inversion is employed with the intention of transformation to temporal domain. In order to examine numerical accuracy of this method, Dirac delta heat flux is applied to the assumed medium and results were compared with those of the analytical solution. It was observed that numerical values follow exact ones, at least up to the seventh order of accuracy. In addition, Step and Triangular heat pulses in the medium were studied to reveal temporal and spatial non-Fourier heat conduction characteristics. It was found that in large values of *Ve* number, for various kinds of heat fluxes carrying the same amount of energy, temperature distribution varies conspicuously through the medium; nevertheless, at each pass of heat wave, a specific point experiences a definite rise of temperature regardless of the type of heat flux provided that the same conditions are present.

1. Introduction

Fourier heat conduction model assumes an infinite propagation speed for thermal disturbances, i.e. when a temperature gradient applies to a medium, everywhere feel it immediately. But in reality, the maximum speed for transportation of a phenomenon is limited to the light speed. Thus, it is clear that disturbances in conduction mode of heat transfer propagate in a finite speed.

However Fourier's assumption may work for many industrial applications; deviation from this model is considerable in many areas in which laser radiation either in medical or industrial purposes, analysis of solar collector plates, applications subjected to high heat fluxes, and heat conduction in the very low ambient temperature are among them [1–4].

Many researches have been carried out to achieve an appropriate non-Fourier heat conduction model which makes up this deviancy and captures experimental evidences in the mentioned areas. The most frequent model was proposed by Cattaneo [5] and Verenotte [6], which simply takes into account the finite speed of heat propagation by means of a first order Taylor series expansion of flux vector in time and consequently adding a lag term in flux equation, as follow:

$$\boldsymbol{q} + \tau \frac{\partial \boldsymbol{q}}{\partial t} = -k\nabla T \tag{1}$$

where **q** is flux vector and τ is thermal relaxation time which depends on the characteristics of employed material. Combination of Eq. (1) and energy equation yields:

$$\frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} = \alpha \nabla^2 T \tag{2}$$

where α is the thermal diffusivity and $C_h = \sqrt{\frac{\alpha}{\tau}}$ is the speed of heat wave propagation. In this way, if $\tau \to 0$, Fourier heat conduction equation will be recovered. It is obvious that Eq. (2) is hyperbolic in nature unlike the classical heat conduction which is parabolic.

Several experimental researches have been conducted to study governing equation of non-Fourier heat conduction and among them, the study of Jackson and Walker [7] on NaF thermal conductivity, second sound and Phonon–Phonon interaction at very low temperature, the observation of second sound in Bismuth by Narayanamurti and Dynes [8], the work of Roetzel et al. [9] on the materials with non-homogeneous inner structure, and more recently the research of HaiDong et al. [10] on the heat conduction in metallic nanofilms from large currents at low temperatures could be mentioned here.

In the other side, numerous studies have been done using analytical and numerical approaches from the earliest ages of introducing governing equation of non-Fourier heat conduction. Considerable quantities of them deal with one-dimensional form of this equation. Tang and Araki [11] studied non-Fourier heat conduction in a finite slab with one isolated boundary and the other subjected to a periodic heat flux by implying Laplace transform analytically. A similar problem is considered by Abdel-Hamid [12] and solved by means of Integral

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Nomenclature

C_h	speed of heat wave propagation
H	Heaviside function
Fo	Fourier number
I_0	modified Bessel function of the first kind of order zero
K	thermal conductivity
L	slab length
N	number of considered terms in the Fourier series expansion
$\mathbf{q}$	flux vector
Q	flux magnitude
S	state variable
T_0	ambient temperature
T	temperature
$\overline{T}$	temperature in state domain
t_0	time of flux applying
t_1	time of flux fading
Ve	Vernotte number

Greek symbols

α	thermal conductivity
σ_0	abscissa of convergence of function
τ	thermal relaxation time
τ_q	relaxation time of flux
τ_t	relaxation time of temperature

Transform Method. Also an analogy between Thermal and Mechanical or Electrical oscillation is shown. In other coordinates, Torabi and Saedodin [13] investigated hyperbolic heat conduction in cylindrical coordinates from either analytical or numerical point of view.

Shirmohammadi and Moosaei [14] studied non-Fourier heat conduction in a hollow sphere with periodic surface heat flux using separation of variables and Duhamel's integral theorem. Recently, Mishra and Sahai studied the implementation of Lattice Boltzmann Method (LBM) to analyze non-Fourier heat conduction in 1-D spherical and cylindrical coordinates and obtained results compared with those from Finite Volume Method [15].

In two or three-dimensional coordinates, because of the existence of the complicated interaction and reflection of thermal waves, conducted researches are not as much as one-dimensional form. Temperature distribution in a rectangular plate with isolated boundaries and constant initial temperature under heat generation is considered by Wu and Chu [16] and heat waves propagation and their refection from the boundaries are studied.

Rahideh et al. [17] studied heat wave propagation in one- and two-dimensional medium subjected to different boundary conditions with temperature-dependent thermal conductivity. Governing equations along with boundary conditions are discretized using Differential Quadrature Method (DQM) and Finite Element Method (FEM) as well. Results are compared, and the effect of various parameters on temperature distribution is studied. Yang [18] introduced a stable finite difference method to solve two-dimensional hyperbolic heat conduction problems and a sequential method to determine boundary condition in inverse problems using a modified Newton–Raphson method at each time step. A three-dimensional case is also considered by Barletta and Zanchini [19], in which a solid bar with rectangular cross section is investigated under hyperbolic heat conduction.

In the above publications, Eq. (1) is used to express flux vector in terms of temperature gradient. But an alternative formulation is available for non-Fourier heat conduction which arises from assuming a first order Taylor series expansion for temperature gradient as well as heat flux. This formulation which is proposed by Tzou [20] is known as dual-phase lag model (DPL) and implies:

$$\boldsymbol{q}+\tau_{\boldsymbol{q}}\frac{\partial \boldsymbol{q}}{\partial t}=-k\left(1+\tau_T\frac{\partial}{\partial t}\right)\nabla T \quad (3)$$

Fan and Lu [21] combined the dual reciprocity boundary element method (DRBEM) with Laplace transforms to attain a numerical solution for Eq. (3) in both one- and two-dimensional form under different boundary conditions. Chou and Yan [22] applied space–time conservation element and solution element (CESE) method to different types of non-Fourier heat conduction, i.e. Single Phase Lag (SPL) with and without heat source as well as DPL, when a finite medium is subjected to surface pulse heating.

In this research, the first type of governing equations is considered due to simplicity, applicability and range of practice. To start with, Laplace Transform is taken from boundary conditions as well as governing equation which led to an ordinary differential equation (ODE) in spatial and state domain. In following, an inverse Laplace Transform is required to achieve the solution in temporal and state domain. But, in point of fact, taking this process analytically is not available in many situations and for many others is considered among the most time-consuming procedures. In the other side, numerical methods taking Laplace transform inversion have been developed rapidly during the last decades. To employ the opportunity given by these methods, a recently developed numerical approach is implemented to find the results in temporal and spatial domains. As it is illustrated in Fig. 1, a one-dimensional slab is considered and a unique heat flux magnitude with different types of time employment i.e. pulse, step, and triangular form, is applied to a boundary and the next one is remained isolated.

The pulse form for flux distribution in time is the toughest and most complicated one. Consequently it could be a reliable reference for numerical procedure validation. Based on this declaration, analytical solution of non-Fourier heat conduction in a one-dimensional finite slab with prescribed boundary conditions by Gembarovic and Majernik [23] is taken, and results of the current numerical approach is compared for validation purposes. After this approach proved to be reliable, further heat flux distributions are applied and results are compared and discussed in detail.

Use of numerical inverse Laplace transform technique to solve non-Fourier heat conduction which gives a high order of accuracy is the first novelty of the present research. Analysis and discussing heat wave behavior in-depth from physical and mathematical point of views, the study of different type of heat fluxes as well as Ve number and their effect on transient temperature distribution, are the next key points studied in this paper as the first turn.

2. Analytical approach

Analytical solution to such problem has been given in previous works done by Gembarovic and Majernik. Solving Eq. (2) in an infinite medium is at hand but for finite slab, it proves to be quite challenging

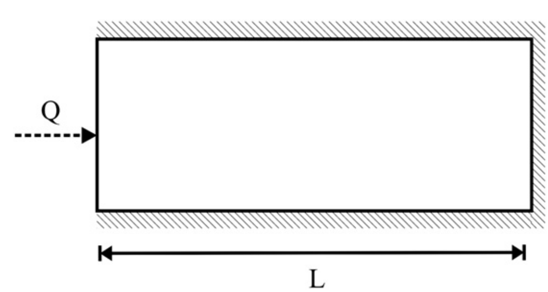

Fig. 1. A two-dimensional slab with insulated boundaries.