International Communications in Heat and Mass Transfer 43 (2013) 75–80

Contents lists available at SciVerse ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

# Flow and heat transfer over an unsteady shrinking sheet with suction in a nanofluid using Buongiorno's model ☆

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ARTICLE INFO

Available online 4 March 2013

*Keywords:*
Buongiorno's model
Boundary layer
Dual solutions
Shrinking sheet
Suction

ABSTRACT

The unsteady flow over a continuously shrinking surface with wall mass suction in a nanofluid using the model proposed by Buongiorno [7] is investigated. By using an appropriate similarity transformation, similarity equations are obtained and the shooting method is used to solve these equations for different values of the wall mass suction, unsteadiness and nanofluid parameters. It is found that dual solutions exist for a certain range of wall mass suction, unsteadiness and nanofluid parameters.



## 1. Introduction

Much attention has been paid to nanofluids due to its enhanced properties and the number of publications related to nanofluids increases in an exponential manner (Ding et al. [1]). Choi [2] made the first attempt to introduce this innovative fluid resulting from the mixture of nanoparticles and the base fluid. The mixture of a base fluid and nanoparticles has unique physical and chemical properties increases the thermal conductivity and therefore substantially enhances the heat transfer characteristics of the nanofluid (Aminossadati [3]). Nanofluids can be defined as the dilution of nanometer-sized particles (smaller than 100 nm) in a fluid (Das et al. [4]), and nanofluids can be produced by dispersing evenly nanoparticles in a base fluid, such as water, ethylene glycol and oil (Wang and Mujumdar [5]). There have been several studies on the mechanism that results in the enhanced heat transfer using nanofluids. Collections of papers on heat transfer in nanofluids can be found in the book by Das et al. [4], and in the review papers by Kakaç and Pramuanjaroenkij [6], Buongiorno [7], and Wang and Mujumdar [8,9]. It is worth mentioning that Buongiorno [7] conducted a study of convective transport in nanofluids with a focus on explaining the heat transfer enhancements which was observed during convective situations. Buongiorno discounts suspension, particle rotation, dispersion, and turbulence as being significant agents which cause heat transfer enhancements. A new model was proposed by Buongiorno [7] and is based on the mechanics of nanoparticles/base-fluid relative velocity. He concluded that Brownian diffusion and thermophoresis dominate when the turbulent effects are absent. He derived the conservation equations based on these two effects.

In this paper, the unsteady flow over a continuously shrinking surface with wall mass suction in nanofluid is numerically studied. We use the nanofluid equations model proposed by Buongiorno [7], as this model has been successfully applied in several papers: Kuznetsov and Nield [10], Khan and Pop [11], Aziz and Khan [12], etc. Similarity equations are obtained by using an appropriate similarity transformation and the corresponding equations are solved numerically using shooting method for different values of wall mass suction parameter, unsteadiness parameter, and nanofluid parameters. We compare our results for the momentum equation with results reported by Fang et al. [13] for a regular (Newtonian) fluid, and they show excellent agreement. It should be pointed out that Miklavčič and Wang [14] has conducted pioneering work on the steady flow over a shrinking sheet. The study of boundary layer flow over shrinking surfaces is an attractive research field and has many practical applications in engineering and industrial processes, for example drawing of plastic films, shrink film, extrusion of polymer sheets from a die, polyester thin wall heat shrink tubing, glass-fiber, paper production and wire drawing, etc. (Abraham and Sparrow [15], Sparrow and Abraham [16], Chauhan and Agrawal [17]). As reported by Goldstein [18], this type of flow is a backward flow and suction is needed to maintain the flow. To the present knowledge of the authors, no studies have been reported in the literature, which investigate the unsteady flow over a shrinking sheet in a nanofluid using the Buongiorno [7] model.

☆ Communicated by Dr. W.J. Minkowycz.

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2013.02.001

## 2. Basic equations

We consider the two-dimensional flow over a continuously unsteady shrinking sheet with mass transfer in a nanofluid. It is assumed that the velocity of the shrinking sheet is $u_w(x, t)$ and the velocity of the mass transfer is $v_w(x, t)$, where $x$ is the coordinate measured along the shrinking sheet and $t$ is the time. It is also assumed that the constant surface temperature and the concentration of the sheet are $T_w$ and $C_w$, while the uniform temperature and concentration far from the sheet are $T_\infty$ and $C_\infty$, respectively. A schematic representation of this problem is shown in Fig. 1.

Assuming that the nanofluid is incompressible and laminar, and using the nanofluid model as proposed by Buongiorno [7], the governing equations of this problem, namely the two dimensional boundary layer equations, are (Kuznetsov and Nield [10]),

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions, respectively, $p$ is the fluid pressure, $T$ is the temperature of the nanofluid, $\nu$ is the kinematic viscosity of the nanofluid, $\alpha$ is the thermal diffusivity of the nanofluid, $C$ is the nanoparticle volume fraction, $D_B$ and $D_T$ are Brownian and thermophoretic diffusion coefficient respectively.

Eqs. (1)–(4) are subjected to the following initial and boundary conditions

$$\begin{aligned}&t<0: u=v=0, T=T_\infty, C=C_\infty \text{ for all } x, y\\ &t\geq 0: u=u_w(x,t)=-\frac{c\,x}{1-\lambda\,t}, v=v_w(x,t), T=T_w, C=C_w \text{ at } y=0\\ &u\to 0, T\to T_\infty, C\to C_\infty \text{ as } y\to\infty\end{aligned} \tag{5}$$

where $c$ is a positive constant and $\lambda$ is a parameter showing the unsteadiness of the physical problem.

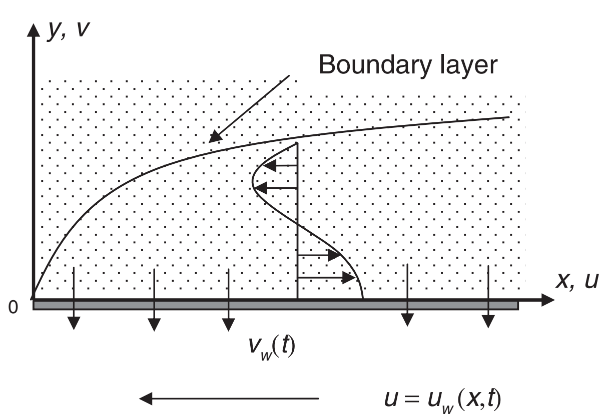


**Fig. 1.** Physical model and coordinate system.

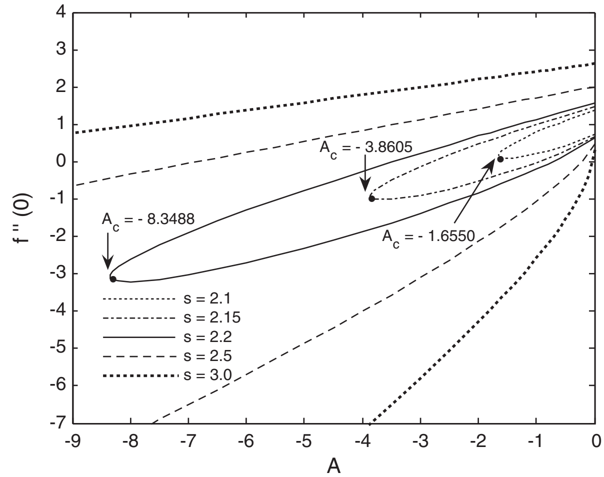


**Fig. 2.** Variation of $f''(0)$ with $A$ for some values of $s$.

We search for a similarity solution of Eqs. (1)–(4) of the following form

$$u=\frac{c\,x}{1-\lambda\,t}f'(\eta),\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}\eta=\sqrt{\frac{c}{\nu(1-\lambda\,t)}}y \tag{6}$$

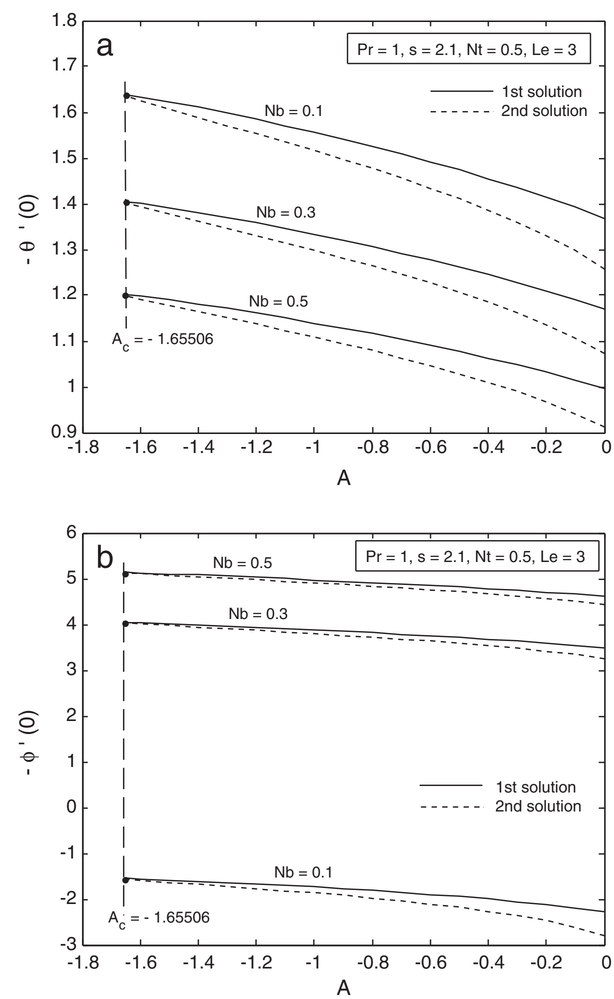


**Fig. 3.** Variation of (a) $-\theta'(0)$ and (b) $-\phi'(0)$ with $A$ for different $N_b$ when $\Pr=1, s=2.1$ and $Le=3$.