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Heatline visualization for conjugate heat transfer of a couple stress fluid from a vertical slender hollow cylinder ☆

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A R T I C L E I N F O

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A B S T R A C T

The flow visualization has been made using the heat function concept for the conjugate heat transfer effects on the transient free convective couple stress fluid flow over a vertical slender hollow circular cylinder with the inner surface kept at a constant temperature. The governing non-linear equations are solved numerically by using an unconditionally stable implicit method. Numerical results show that the deviations of flow variables of couple stress fluid from those of the Newtonian fluid turn out to be considerable. Boundary layer flow visualization indicates that the streamlines exist starting from the leading edge to the far downstream, while the heatlines terminate at a finite distance from the cylinder wall. It is noticed that the steady-state values of average skin-friction and heat transfer rate decrease as the conjugate-conduction parameter increases.

1. Introduction

Unsteady natural convective flow of a viscous incompressible fluid is an important problem relevant to many engineering applications. In the glass and polymer industries, hot filaments are considered as vertical cylinders and cooled as they pass through the surrounding environment. In most of these situations, the temperature distribution in the fluid is mutually coupled with the temperature distribution in the solid body over which the fluid flows. These types of problems are studied in-depth in the literature. It can be observed that in the previous investigations the wall conduction resistance in the case of convective heat transfer between a solid cylinder wall and a fluid flow is generally neglected i.e. the wall is assumed to be very thin [1,2]. But in many practical problems the information on the interfacial temperature is essential because the heat transfer characteristics are mainly determined by the temperature differences between the bulk flow and the interface. In order to take the account of physical reality, there has been a proclivity to move away from considering idealized mathematical problems in which the bounding wall is considered to be infinitesimally thin. Thus the conduction in solid wall and the convection in the fluid should be determined simultaneously. This type of convective heat transfer is referred to as a conjugate heat transfer (CHT) process and it arises due to the finite thickness of the wall. These types of problems have many practical applications, particularly those related to energy conservation in buildings, cold storage installations and cryogenic applications, such as medical and space technology and are studied extensively [3–5].

Also, with the growing importance of non-Newtonian fluids in modern technology and industries, the investigations on such fluids are desirable. Stokes [6] generalized the classical Newtonian model to include the effect of couple stresses in a way different from that of Eringen [7]. This is one among the several non-Newtonian fluid theories that are developed in the twentieth century. In his theory Stokes considered a body enclosing a volume without considering the microstructures of the infinitesimal fluid volume element. The set of all forces acting on an infinitesimal volume element is, in general, assumed to be equivalent to a single resultant force together with a resultant couple. The moment of the couple is assumed to be of non-zero value. With this assumption Stokes has proposed the theory of couple stress fluids allowing for the sustenance of couple stresses in addition to the usual stresses. Also, in his theory, curvature twist rate tensor is proposed based on the pure kinematic aspects of rotation vector and couple stress is defined in terms of this curvature twist rate tensor. Accordingly, in the balance of linear momentum of the couple stress flow model, fourth order derivatives of velocities are involved and, hence, separate angular momentum equation need not be considered. These fluids can also sustain the existence of body forces as usual and in addition to the body couples as well. The stress tensor is no longer symmetric in this theory. The fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, such as blood, lubricants containing small amount of polymer additive, electro-rheological fluids and synthetic fluids are some of the examples for these couple stress fluids. This couple stress model has been widely used because of its great mathematical simplicity compared to that of the other models developed for the polar fluids. Recently, the study of couple stress fluid flows has been the subject of great interest, due to its widespread industrial and scientific applications as in the case of micropolar fluids. Important field where couple stress fluids have applications includes squeezing and lubrication [8],

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Nomenclature

$\overline{C_f}$	dimensionless average skin-friction coefficient
c_p	specific heat at constant pressure
d_{ij}	rate of deformation tensor
g	acceleration due to gravity
Gr	Grashof number
k_f, k_s	thermal conductivity of the fluid and the solid cylinder, respectively
l	length of the cylinder
m	trace of couple stress tensor
m_{ij}	couple stress tensor
$\overline{Nu}$	average Nusselt number
P	conjugate-conduction parameter
p	fluid pressure
Pr	Prandtl number
r	radial coordinate
r_i, r_0	inner and outer radii of the hollow cylinder, respectively
R	dimensionless radial coordinate
t'	time
t	dimensionless time
t_{ij}	force stress tensor
T_0	temperature at the inside surface of the cylinder
T'	temperature of the fluid
u, v	velocity components in x, r directions, respectively
U, V	dimensionless velocity components in X, R directions, respectively
$\mathbf{U}$	velocity vector
x	axial coordinate
X	dimensionless axial coordinate

Greek Letters

α	thermal diffusivity
β	volumetric coefficient of thermal expansion
δ_{ij}	Kronecker delta
ε_{ijk}	Levi-Civita symbol
η, η'	couple stress viscosity coefficients
μ, λ	viscosity coefficients
$\omega_{i,j}$	spin tensor
ω	spin vector
Π'	heat function
Π	dimensionless heat function
ψ	dimensionless stream function
ρ	density
θ	dimensionless temperature of the fluid
ν	kinematic viscosity

Subscripts

w	conditions on the wall
∞	free stream conditions

bio-fluid mechanics [9], MHD flows and synthesis and plasticity of chemical compounds.

From the literature survey, it can be noted that the CHT on the unsteady natural convective flow of a viscous incompressible couple stress fluid past a vertical cylinder has received very scant attention in the literature. Hence, in the present investigation our attention is focused on the CHT problem of transient free convection over the outside surface of a vertical slender hollow cylinder. In general, studies on natural convection have been carried out with streamlines and isotherms. Note that, isotherms are generally used to illustrate the temperature distribution in a domain respectively, however, isotherms may not be suitable to visualize the direction and intensity of heat transfer particularly in convection problems in which the path of heat flux is not perpendicular to isotherms due to convection effect. When dealing with two-dimensional fluid flows, it is not the isobars but the streamlines that are the best tools for visualization and analysis, as fluid flows are not in the direction perpendicular to the isobars [10]. Similarly, when dealing with the two-dimensional convective heat transfer, it is not the isotherms but the heatlines that are the best tools for visualization and analysis. The main use of heatlines is to find the flow intensity in the region which is not observed in the other contours such as velocity and temperature. The heatlines are the more adequate tools for visualization and analysis of heat transfer process, giving well defined corridors where energy transfer occurs from the hot wall to the cold wall. The heatlines are mathematically represented by heat functions and the proper dimensionless forms of heat functions are closely related to the overall Nusselt numbers. The heatline concept was first introduced by Kimura and Bejan [11] and Bejan [12]. A detailed review on applications of heatlines and masslines was also performed by Costa [10]. The use of heatlines in the unsteady problems was first studied by Aggarwal and Manhapra [13,14] to analyze the unsteady heat transfer process in cylindrical enclosures subjected to natural convection. Recently, Basak et al. [15] studied the analysis of heatlines within triangular cavities. Till date, the heatline concept has been paid less attention for analyzing convective heat transfer processes except for very few applications. Based on this literature survey, an attempt is made for the first time to study the concept of heatlines for the present investigated problem.

In Section 2, description of the problem is given and the corresponding governing equations are derived. The details about the numerical method and the grid generation are explained. The average skin-friction and average heat transfer rate are also derived. In addition, heat function has been derived and non-dimensionalized based on the overall average Nusselt number on the hot wall. In Section 3, the comparison between the couple stress fluid flow and Newtonian fluid flow is analyzed. The average values of skin-friction and heat transfer rate with respect to time are shown graphically and discussed. Also the visualization of streamlines, isotherms and heatlines is shown. Finally, the concluding remarks are made in Section 4.

2. Mathematical formulation and simulation

A natural convective couple stress fluid flow past a vertical slender hollow cylinder of length l and the outer radius r_0 ($l >> r_0$) is considered as shown in Fig. 1. The x-axis is measured vertically upward along the axis of the cylinder. The origin of x is taken to be at the leading edge of the cylinder, where the boundary layer thickness is zero. The radial coordinate, r, is measured perpendicular to the axis of the cylinder. It is assumed that the fluid has constant physical properties and the fluid flow is unsteady, laminar and two-dimensional. The surrounding stationary fluid temperature is assumed to be of ambient temperature $\left(T'_\infty\right)$. The temperature of the inside surface of the cylinder is maintained at a constant temperature of T'_0, where $T'_0 > T'_\infty$. Initially, i.e., at time $t' = 0$ it is assumed that the outer surface of the cylinder and the fluid are of the same temperature T'_∞. As time increases ($t' > 0$), the temperature of the outer surface of the cylinder is raised to the solid-fluid interface temperature T'_w and maintained at the same level for all time $t' > 0$. It is assumed that the effect of viscous dissipation is negligible in the energy equation.

2.1. Governing equations

Based on the above assumptions and Boussinesq's approximation, the flow of an incompressible couple stress fluid in the absence of