International Communications in Heat and Mass Transfer 48 (2013) 164–177

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Lattice Boltzmann simulation of natural convection heat transfer in an elliptical-triangular annulus☆

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ARTICLE INFO

Available online 7 September 2013

Keywords:
Lattice Boltzmann method (LBM)
Free convection
Triangular cylinder
Elliptical cylinder
Curve boundary
Annuli

ABSTRACT

A numerical study for steady-state, laminar natural convection in a horizontal annulus between a heated triangular inner cylinder and cold elliptical outer cylinder was investigated using lattice Boltzmann method. Both inner and outer surfaces are maintained at the constant temperature and air is the working fluid. Study is carried out for Rayleigh numbers ranging from 1.0×10^3 to 5.0×10^5. The effects of different aspect ratios and elliptical cylinder orientation were studied at different Rayleigh numbers. The local and average Nusselt numbers and percentage of increment heat transfer rate were presented. The average Nusselt number was correlated. The results show that by decreasing the value of aspect ratio and/or increasing the Rayleigh number, the Nusselt number increases. Also the heat transfer rate increases when the ellipse positioned vertically.

1. Introduction

Natural convection is one of the simplest mechanisms of heat transfer and has been used in many applications for purpose of cooling or heating in industry. The annular shape enclosure is one of the applicable geometry in engineering and industry; the geometry of the horizontal circular annulus is commonly found in solar collector-receiver, underground electric transmission cables and vapor condenser for water distillation, heat exchangers and food processing equipment. In the past, many studies were performed about natural convection in a horizontal annulus between two circular cylinders. Kuehn and Goldstein [1,2] reviewed the most reliable studies about the free convection in the annulus. Also, they conducted an experimental and theoretical analyzes of natural convection in concentric and eccentric horizontal cylindrical annuli. Vertical and horizontal eccentricity of the inner cylinder was examined by Glakpe et al. [3] and Guj and Stella [4] at constant heat flux and isothermal boundary conditions, respectively. Fewer studies were investigated natural convection in the non-circular cylinder, such as elliptical annuli and triangular cylinder as an inner or outer cylinder.

Theoretical and experimental studies of steady-state natural convection in a symmetric annulus space between two concentric, confocal elliptic tubes were performed by Lee and Lee [5]. In the investigation, the natural convection formulation was developed in elliptical coordinate system and solved by finite difference method (FDM), and the results were compared with experimental data.

Elshamy et al. [6] performed numerical simulations of free convection in a horizontal confocal elliptical annulus. Finite volume method (FVM) was used to simulate the problem, and the results were reported for local and average Nusselt numbers. Numerical investigations of buoyancy driven flows in horizontal concentric and eccentric elliptical geometry were carried out. The governing equation was discretized by body-fitted curvilinear coordinate transformation method and solved by finite volume method. Zhu et al. [7] utilized the differential quadrature (DQ) method to simulate steady natural convection in a symmetrical elliptic annuli shape. Mahfouz and Badr [8] investigated transient and steady natural convection heat transfer in an elliptical annulus. They used Fourier spectral method to solve mass, momentum and energy equations. Velocity and thermal fields and average Nusselt number were presented and discussed.

Natural convection in triangular enclosures was investigated by many authors [9,10]. Triangle inner or outer cylinder was examined by Xu et al. [11,12]. They used finite volume approach to solve the governing equations for laminar natural convective heat transfer from a horizontal triangular cylinder to concentric cylindrical enclosure. The work examined different radius ratios and inclination angles for the inner triangular cylinder, and results were presented in term of streamlines, temperature contours and Nusselt number. They also used a horizontal cylinder inside a concentric triangular enclosure and examined the effect of different cross section of the inner cylinder.

The lattice Boltzmann method is a numerical method that has been recently developed and used in simulation of complex phenomena in fluid mechanics [13] such as conjugate heat transfer[14,15], nonfluid

☆ Communicated by W.J. Minkowycz.

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2013.08.009

Nomenclature

a	length of semi-major axis of ellipse [m]
Ar	axis ratio of ellipse, $\mathrm{Ar} = b/a$ [−]
b	length of semi-minor axis of ellipse [m]
BR	aspect ratio, $\mathrm{BR} = b/R$ [−]
c	discrete lattice velocity [$\mathrm{m^2/s}$]
f	distribution function for density [−]
F	total body force [−]
g	distribution function for temperature [−]
g_y	acceleration due to the gravity in $(-y)$ direction [$\mathrm{ms^{-2}}$]
L	characteristic length [m]
Nu	Nusselt number [−]
P	Perimeter of triangle [m]
Pr	Prandtl number, $\Pr = \upsilon/\alpha$ [−]
R	radius of the circumscribed circle of the equilateral triangle, [m]
Ra	Rayleigh number, $Ra = g\beta\Delta T(a-R)^3/\alpha\upsilon$ [−]
T	temperature [K]
t	time [s]

$\vec{u} = \left(u\,\vec{i} + v\,\vec{j}\right)$
velocity [m/s]

$\vec{x} = \left(x\,\vec{i} + y\,\vec{j}\right)$
location vector

Greek symbols

α	thermal diffusivity coefficient [$\mathrm{m^2/s}$]
β	thermal expansion coefficient [1/°C]
Δt	lattice time step
ε	dimensionless eccentricity of ellipse, $\varepsilon = \sqrt{1-Ar^2}$ [−]
$\delta = \frac{\lvert x_f - x_w \rvert}{\lvert x_f - x_b \rvert}$	fraction of an intersected link in the fluid region [−]
ρ	density [$\mathrm{kg/m^2}$]
τ_t	relaxation time for temperature equation [−]
τ_v	relaxation time for velocity equation [−]
υ	kinematic viscosity [$\mathrm{m^2/s}$]
χ	weighting factor [−]
ω	weighting factor [−]
$\theta = \frac{T - T_c}{T_h - T_c}$	dimentionless temperature [−]

Subscripts

f	first fluid nodes
ff	second fluid nodes
b	boundary nodes
w	wall nodes
avg	average
c	cold
eff	effective
h	hot
in	inner cylinder
k	lattice model direction
l	local
out	outer cylinder
s	sound

Superscript

eq	equilibrium distribution function
neq	non-equilibrium distribution function
~	post-collision state of the distribution function.

[16,17] and porous media [18]. In comparison with the conventional CFD methods, the advantages of LBM include simple calculation procedure, simple and efficient implementation for parallel computation, easy and robust handling of complex geometries, etc. Various numerical simulations have been performed using different thermal LB models or Boltzmann-based schemes to investigate the natural convection problems [19–22].

Numerical investigation of natural convection heat transfer in a horizontal concentric annulus using lattice Boltzmann simulation was performed by Shi et al. [23]. They proposed a Finite difference-based LBGK model for heat transfer based on the discrete velocity model in curvilinear coordinates.

Fattahi et al. [24] simulated natural convection heat transfer in eccentric annulus using lattice Boltzmann model (LBM) based on double-population approach. They successfully used an extrapolation method based on Guo et al. [25] and Mei et al. [26] velocity boundary condition approach. Also they investigated mixed convection heat transfer in an eccentric annulus based on multi-distribution function, double-population approach [27]. Osman et al. [28] investigated natural convection from a concentrically and eccentrically inner heated cylinder placed inside a cold outer cylinder. To solve the problem, a double-population thermal lattice Boltzmann was used. The D2Q4 and D2Q9 BGK models were selected to determine the temperature and velocity fields, respectively.

The aim of the present study was a laminar natural convection heat transfer between the concentric inner heated triangular cylinder and outer elliptical enclosure using the lattice Boltzmann method (Fig. 1(a)). The study has been carried out for Rayleigh number in the range of $10^3 < Ra < 5 \times 10^5$, where the Prandtl number fixed at 0.71, and the aspect ratio was changed from 1.2 to 3. Correlations were suggested for average Nusselt number at different Rayleigh numbers and shape configurations.

2. Lattice Boltzmann method

In contrast to the classical macroscopic Navier–Stokes (NS) approach, the lattice Boltzmann method (LBM) uses a mesoscopic model to simulate fluid flows. The LBM is a discretization of Boltzmann's transport equation (BTE). To develop the BTE, Boltzmann assumed a fluid made of particles that collide according to the laws of classical mechanics. In LBM, the domain is discretized in uniform Cartesian cells which each one holds a fixed number of distribution functions, which represent the number of fluid particles moving in discrete directions. For present work, the D2Q9 model is used, which consists of nine distribution functions. The distribution functions are calculated by solving the lattice Boltzmann equation (LBE), which is a special discretization of the kinetic Boltzmann equation. The LBM is based on simultaneously solving two distribution functions, one for velocity field and the other for the temperature field [29]. After introducing BGK (Bhatnagar–Gross-Krook) approximation, the Boltzmann equation without external forces can be formulated as below [30]:

$$\frac{\partial f}{\partial t} + \vec{c} \cdot \vec{\nabla} f = \frac{1}{\tau_v}\left(f^{eq} - f\right) \tag{1}$$

In this equation, f is the distribution function. In the lattice Boltzmann method, Eq. (1) is discretized and assumed valid along specific directions and linkages. Therefore, the discrete Boltzmann equation can be written as

$$f_k\left(\vec{x} + \vec{c}_k \Delta t, t + \Delta t\right) - f_k\left(\vec{x}, t\right) = \Delta t \frac{f_k^{eq}\left(\vec{x}, t\right) - f_k\left(\vec{x}, t\right)}{\tau_v} \tag{2}$$