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Analytical device-physics framework for non-planar solar cells

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ABSTRACT

Non-planar solar cell devices have been promoted as a means to enhance current collection in absorber materials with charge-transport limitations. This work presents an analytical framework for assessing the ultimate performance of non-planar solar cells based on materials and geometry. Herein, the physics of the *p-n* junction is analyzed for low-injection conditions, when the junction can be considered spatially separable into quasi-neutral and space-charge regions. For the conventional planar solar cell architecture, previously established one-dimensional expressions governing charge carrier transport are recovered from the framework established herein. Space-charge region recombination statistics are compared for planar and non-planar geometries, showing variations in recombination current produced from the space-charge region. In addition, planar and non-planar solar cell performances are simulated, based on a semi-empirical expression for short-circuit current, detailing variations in charge carrier transport and efficiency as a function of geometry, thereby yielding insights into design criteria for solar cell architectures. For the conditions considered here, the expressions for generation rate and total current are shown to universally govern any solar cell geometry, while recombination within the space-charge region is shown to be directly dependent on the geometrical orientation of the *p-n* junction.

1. Introduction

Contemporary solar cell designs are based on a planar geometry, for which charge carrier separation and photon absorption are approximately one-dimensional within the device. Using one-dimensional physics for simulating device performance is generally appropriate for most planar devices, when the solar cell is approximated as a simple, 1D, *p-n* junction. There are cases, however, such as when considering high-efficiency solar cell designs, (*e.g.* rear point-contact and/or back-contact solar cells), that using one-dimensional physics may not truly capture the intricacies of the device, and 2D/3D finite element methods are typically used to simulate performance. In addition to factors such as material and manufacturing costs [1–3], novelty of non-planar solar cell architectures is grounded in the idea that some non-planar devices decouple optical and electronic path lengths [4] and, therefore, offer opportunities to alter the competing roles of charge carrier collection and recombination within a device, which limit efficiency for planar cells with low charge carrier mobility and lifetime. In the last decade, a number of unconventional, non-planar solar cell designs have been proposed, and some experimentally fabricated [4–13], in efforts to increase energy conversion efficiencies. To date, however, the "planar" solar cell architecture still holds all efficiency records over its non-planar counterparts [14]. While there is substantial information describing analytical 1D device physics of planar solar cells [15–24], a comparatively small amount of literature is available describing analytical charge carrier transport properties for non-planar solar cell devices [14,25–30]. By detailing the device physics of a "geometrically generalized" solar cell, devices of various geometrical architectures are modeled congruently to ascertain the conditions under which non-planar configurations improve efficiency.

Our aim is to develop a simple framework for analytically calculating solar cell current as a function of voltage for a geometrically generalized *p-n* junction solar cell, analogous to the model employed to analytically calculate current as a function of voltage for a planar *p-n* junction solar cell. The purpose in doing so is to provide a better practical understanding of charge carrier transport for non-planar solar cells, and to explain how geometry, alone, can easily, and significantly, alter solar cell performance. Previous attempts to compare device performance of cylindrical/radial and planar solar cells have focused on minority charge carrier diffusion in the quasi-neutral regions (QNR's) [26–30]. As such, we do not attempt to improve upon previous efforts solving for minority charge carrier diffusion current densities in the QNR's and, instead, focus on developing a generalized curriculum for the constituent components that govern solar cell *I-V* characteristics of non-planar geometries.

In the low injection limit, a *p-n* junction is typically considered to be spatially separable into two QNR's and a space-charge region (SCR). We display a spatially generalized *p-n* junction in Fig. 1, with

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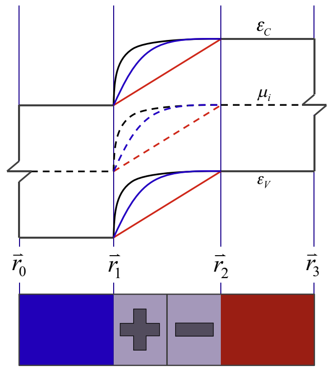

Fig. 1. Generalized *p-n* junction energy band diagram in the low injection limit. The spatial energy dependence is shown for planar (linear dependence; red), cylindrical (logarithmic dependence; blue), and spherical (inverse dependence; black) architectures.

the distinct regions indicated by vector positions $\vec{r}_i$, $i=0,1,2,3$, for the purpose of indicating the spatial orientation of the junction in this discussion. In addition, we also show proposed energy band modifications in the SCR, which we explain in further detail in subsequent sections. In our analysis, a given transport variable $X(r)$ in the n-type QNR (nQNR) is implied by the relationship $X(\vec{r}_0 \le \vec{r} \le \vec{r}_1) \equiv X_N(\vec{r})$, based on the p-n junction shown in Fig. 1. Likewise, in the SCR, $X(\vec{r}_1 \le \vec{r} \le \vec{r}_2) \equiv X_{SC}(\vec{r})$, and in the p-type QNR (pQNR), $X(\vec{r}_2 \le \vec{r} \le \vec{r}_3) \equiv X_P(\vec{r})$.

2. Theory

2.1. Total device current

The area over which charge extraction occurs for a *p-n* junction is a function of position and, therefore, is not necessarily uniform for non-planar devices. Thus, current *density* (*i.e.* charge per unit time per unit area) is not necessarily conserved for all solar cell architectures, though it certainly is for the planar geometry [15–17,20–24]. However, current (*i.e.* charge per unit time) is fundamentally conserved for all geometries. Therefore, we deviate from traditional methods attempting to model total current density of a solar cell, and instead focus on calculating total current, because it is more fundamental to non-planar solar cell performance. To arrive at an expression for total current of the geometrically generalized solar cell device, we apply conservation of current at a specific position along the *p-n* junction, analogous to the methodology applied to planar solar cells. However, because manipulation of the drift-diffusion and continuity equations in the QNR's yield expressions for current density, not current, we write conservation of current at a specific position in the *p-n* junction in terms of area integrals over current density; *i.e.*

$$i_{total} = \iint \vec{j}_n\left(\vec{r}\right) \cdot d\vec{a}\left(\vec{r}\right) + \iint \vec{j}_p\left(\vec{r}\right) \cdot d\vec{a}\left(\vec{r}\right). \tag{1}$$

According to the planar analysis [15–17,20–24], appropriate positions along the solar cell device to sum the electron and hole current densities are at either $\vec{r}=\vec{r}_1$ or $\vec{r}=\vec{r}_2$, as these positions share boundaries with the SCR. By combining conservation of current density with a charge carrier continuity equation across the SCR, the expression for total current density becomes a sum of minority charge carrier current densities from the QNR's, evaluated at the edges of the SCR (positions $\vec{r}=\vec{r}_1$ and $\vec{r}=\vec{r}_2$), and generation and recombination current densities from across the SCR [15–17,20–24]. Here, we employ the same methodology, but now apply it to conservation of current in Eq. (1). In addition, we re-write Eq. (1) in terms of generalized coordinates, so that the expression for total current may be utilized by any coordinate system. In this way, the expression for total current is universal for all geometrical orientations of a *p-n* junction, provided that the junction is established symmetrically along only one axis of a coordinate system (*i.e.* current density is flowing parallel to only one unit vector normal of an area element), and that the low-injection limit is applicable. For a three dimensional system of generalized coordinates q_i, the position vector $\vec{r}$ is defined by

$$\vec{r} = \sum_{i=1}^{3} q_i \hat{e}_i \,, \tag{2}$$

the gradient $\vec{\nabla}$ is defined by

$$\vec{\nabla} A = \sum_{i=1}^{3} \frac{1}{h_i\left(\vec{r}\right)} \frac{\partial A}{\partial q_i} \hat{e}_i \tag{3}$$

for a scalar A, divergence is defined by

$$\vec{\nabla} \cdot \vec{B} = \frac{1}{h_1\left(\vec{r}\right) h_2\left(\vec{r}\right) h_3\left(\vec{r}\right)} \left[\frac{\partial}{\partial q_1}\left[h_2\left(\vec{r}\right) h_3\left(\vec{r}\right) B_1 \right] + \frac{\partial}{\partial q_2}\left[h_1\left(\vec{r}\right) h_3\left(\vec{r}\right) B_2 \right] + \frac{\partial}{\partial q_3}\left[h_1\left(\vec{r}\right) h_2\left(\vec{r}\right) B_3 \right] \right] \tag{4}$$

for a vector $\vec{B}$, and the infinitesimal area element $d\vec{a}\left(\vec{r}\right)$ is defined by

$$da\left(\vec{r}\right)\,\hat{n} = \sum_{i=1}^{3} da_i\left(\vec{r}\right)\,\hat{e}_i. \tag{5}$$

For all expressions, the elements $\hat{e}_i$ represent unit vectors, and $h_i(\vec{r})$ represent the coordinate transformation factors (*e.g.* in cylindrical coordinates, $h_1(\vec{r})=1$, $h_2(\vec{r})=\rho$, and $h_3(\vec{r})=1$).

For conservation of current evaluated at $\vec{r}=\vec{r}_1$, Eq. 1 becomes

$$i_{total} = \iint \vec{j}_{p_N}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_1} . d\vec{a}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_1} + \iint \vec{j}_{n_N}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_1} . d\vec{a}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_1}. \tag{6}$$

However, from the low injection analysis of the drift-diffusion and charge continuity equations in the QNR's, no expression for the *majority* electron current density $\vec{j}_{n_N}(\vec{r})$ in the nQNR is readily available; only *minority* charge carrier expressions are readily available in the QNR's. The planar analysis circumnavigates this issue by determining the *majority* electron current density at $\vec{r}=\vec{r}_1$ in terms of the *minority* electron current density at $\vec{r}=\vec{r}_2$ [15–17,20–24]. To determine the electron current density at $\vec{r}=\vec{r}_1$, ($\vec{j}_{n_N}(\vec{r})|_{\vec{r}=\vec{r}_1}$) in terms of the electron current density at $\vec{r}=\vec{r}_2$, ($\vec{j}_{n_P}(\vec{r})\big|_{\vec{r}=\vec{r}_2}$) the *electron* continuity equation is integrated across the SCR, which is equally valid for non-planar *p-n* junctions when assuming electron flow along only one coordinate axis q_1; *i.e.*

$$h_2\left(\vec{r}\right) h_3\left(\vec{r}\right) \vec{j}_{n_P}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_2} - h_2\left(\vec{r}\right) h_3\left(\vec{r}\right) \vec{j}_{n_N}\left(\vec{r}\right)\Big|_{\vec{r}=\vec{r}_1} = -q \int_{\vec{r}_1}^{\vec{r}_2} h_1\left(\vec{r}\right) h_2\left(\vec{r}\right) h_3\left(\vec{r}\right) \left[G_{SC}\left(\vec{r}\right) - U_{SC}\left(\vec{r}\right) \right] dq_1 \tag{7}$$