International Communications in Heat and Mass Transfer 40 (2013) 1–6

Contents lists available at SciVerse ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

MHD viscoelastic fluid non-Darcy flow over a vertical cone and a flat plate ☆

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ARTICLE INFO

Available online 13 November 2012

Keywords:
MHD
Viscoelastic fluid
Non-Darcy porous medium
Double dispersion effects
Soret effect
Dufour effect

ABSTRACT

A study has been carried out to analyze the double dispersion effects on unsteady, free convective, chemically reacting, MHD viscoelastic fluid (Walters liquid-B model) flow over a vertical cone and a flat plate saturated with non-Darcy porous medium in the presence of Soret and Dufour effects. The constitutive equations for the boundary layer regime are solved by an efficient finite difference scheme of the Crank-Nicolson type. The features of the fluid heat and mass transfer characteristics are analyzed by plotting graphs and the physical aspects are discussed in detail to interpret the effect of significant parameters of the problem. The overall heat and mass transfer profiles are enhanced for increasing the thermal and solutal dispersion effects, respectively. The results indicate that the Soret and Dufour effects have considerable effect on the viscoelastic fluid flow through non-Darcy porous medium.

1. Introduction

The use of viscoelastic fluids has grown considerably because of many applications in chemical process industries, biosystems, food processing and biomedical engineering. The Walters-B viscoelastic fluid model was developed by Walters [1] to simulate the viscous fluids possessing short memory elastic effects and can simulate accurately many complex polymeric, biotechnological and tribological fluids. Mohiddin et al. [2] studied the flow, heat and mass transfer characteristics of Walters-B viscoelastic fluid flow over a vertical cone. In a recent investigation, Chang et al. [3] have shown interest to analyze the Walters-B viscoelastic fluid flow through vertical plate.

In the last few decades, heat and mass transfer analysis of chemically reacting fluid flow through porous medium with the influence of magnetic field has attracted considerable attention of researchers because such process exists in many branches of science and technology [4,5]. In many practical situations, the porous medium is bounded by an impermeable wall, has high flow rates, and reveals non-uniform porosity distribution in the near wall region, making the Darcy's law inapplicable. So it is necessary to include the non-Darcian effect in the analysis of convective transport in porous medium. The inertia effect is expected to be important at the higher flow rate and it can be accounted through the addition of a velocity squared term in the momentum equation, which is known as Forchheimer's extension [6,7]. It is known from the literature that the thermal dispersion effect will become significant in the non-Darcy medium when the inertia effect is substantial [8]. Mixing and recirculation of local fluid streams occur as the fluid moves through tortuous paths in packed beds. This hydrodynamic mixing of fluid at pore level causes the dispersion effects in porous medium. This becomes more considerable for moderate and fast flows. The development of dispersion theory has been mainly related to miscible displacement and solute spreading in porous media. These areas are-of major interest to secondary and tertiary oil recovery operations and to pollution control in water resources engineering. The influence of thermal dispersion on non-Darcy convection over a cone was investigated by Murthy and Singh [9]. Detailed discussion and literature survey of the thermal and solutal dispersion effects on plate are available in the Refs. [10,11].

Soret and Dufour effects are considered as second order phenomena, on the basis that they are of smaller order of magnitude than the effects described by Fourier's and Fick's laws, and often neglected in many heat and mass transfer studies. However, the importance of Soret effect is becoming more widely accepted; examples are in geosciences when considering magma differentiation, in petrology when investigating hydrocarbon segregation. The Soret effect has also been utilized for isotope separation and in mixtures between gasses with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air). In most liquid mixtures, the Dufour effect is inoperative, but this may not be the case in gasses [12]. Cheng [13] had studied and reported the significance of Soret and Dufour effects on natural convection heat and mass transfer from a vertical cone in a porous medium. Postelnicu [14] had investigated the heat and mass transfer characteristics over a plate with the influence of Soret and Dufour effects. Some interesting contributions pertaining to fluid flow and heat transfer characteristics from a cone and a wedge are found in [15,16].

☆ Communicated by W.J. Minkowycz.

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http://dx.doi.org/10.1016/j.icheatmasstransfer.2012.10.025

Nomenclature

u	component of dimensional velocity along x direction
v	component of dimensional velocity along y direction
r	dimensional local radius of the cone
t^*	dimensional time
K_0	dimensional Walters-B viscoelasticity parameter
C_b	drag coefficient which is independent of viscosity
K_1	dimensional porous permeability parameter
d	pore diameter
k	thermal conductivity of the fluid
K_T	thermal diffusion ratio
C_P	specific heat at constant pressure
C_S	concentration susceptibility
D	constant molecular diffusivity
D_d	dispersion molecular diffusivity
T_m	mean fluid temperature
E	Walters-B viscoelastic parameter
K	permeability coefficient of porous medium
Gr_T	thermal Grashof number
N	buoyancy ratio parameter
M	magnetic field parameter
F_I	local inertia coefficient
P_r	Prandtl number
D_u	Dufour number
S_c	Schmidt number
K_r	chemical reaction parameter
S_r	Soret number

Greek symbols

σ	electric conductivity of the fluid
β_T	thermal expansion coefficient
β_C	concentration expansion coefficient
α_0	constant thermal diffusivity
α_d	dispersion thermal diffusivity
γ^*	dimensional thermal dispersion coefficient
ξ^*	dimensional solutal dispersion coefficient
α	semi vertical angle of the cone
γ	thermal dispersion coefficient
α_T	heat source/sink parameter
ξ	solutal dispersion coefficient

In view of the above discussions, authors envisage to investigate the double dispersion effects on MHD viscoelastic fluid non-Darcy flow over a vertical cone and a flat plate with the influence of Soret and Dufour effects which have immaculate applications in fields like ceramics production, filtration and oil recovery etc. The Crank–Nicolson method is used to solve the coupled nonlinear equations of the problem. The results of parametric study on the temperature, concentration, Nusselt number and Sherwood number distributions are shown graphically and the physical aspects are discussed in detail.

2. Mathematical formulation

We consider a two-dimensional, unsteady, free convective flow of an incompressible viscoelastic (Walters liquid-B model) fluid over a vertical cone and a flat plate saturated with non-Darcy porous medium as shown in Fig. 1.

The x-axis is taken in the direction along the surface of the cone and plate which is set in motion and the y-axis is taken perpendicular to it. The wall $y=0$ is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞ and

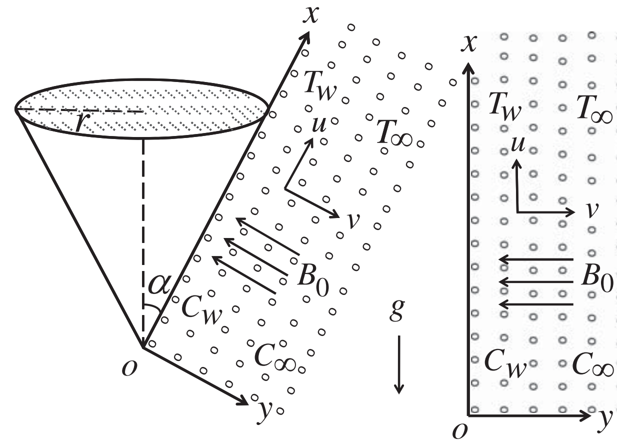

Fig. 1. Flow geometry of the problem.

ambient concentration C_∞, respectively. The fluid is electrically conducting in the presence of an external transversely applied, uniform magnetic field of strength B_0. The magnetic Reynolds number is assumed to be very small so that both the Hall effects and the induced magnetic field are negligible. The flow field exposed the influence of buoyancy effect. The heat equation includes the thermal dispersion, heat source/sink and diffusion-thermo effects whereas the mass transfer equation includes the effects of solutal dispersion, first order chemically reactive species and thermal-diffusion. Taking into consideration of these assumptions, the equations that describe the physical situation can be written in the Cartesian frame of references, as follows:

$$\frac{\partial\left(r^h u\right)}{\partial x}+\frac{\partial\left(r^h v\right)}{\partial y}=0 \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=\nu\frac{\partial^2 u}{\partial y^2}-K_0\frac{\partial^3 u}{\partial t^*\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{C_b}{\sqrt{K_1}}u^2\\&-\frac{\nu}{K_1}u+g\beta_T cos(\alpha)(T-T_\infty)+g\beta_C cos(\alpha)(C-C_\infty)\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t^*}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right)+\frac{Q_T}{\rho C_P}(T-T_\infty)+\frac{DK_T}{C_S C_P}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t^*}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right)-K_R(C-C_\infty)+\frac{DK_T}{T_m}\frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The appropriate initial and boundary conditions of the problem are

$$\begin{aligned}&t^*\le 0: u=0, v=0, T=T_\infty, C=C_\infty \quad \text{for} \quad \text{all}\, x, y\\&t^*>0: u=0, v=0, T=T_w, C=C_w \quad \text{at} \quad y=0\\&\qquad u=0, T=T_\infty, C=C_\infty \quad \text{at} \quad x=0\\&\qquad u\to 0, T\to T_\infty, C\to C_\infty \quad \text{as} \quad y\to\infty\end{aligned} \tag{5}$$

where $h=1$ corresponds to flow over a vertical cone and $h=0$ with $\alpha=0$ corresponds to flow over a vertical flat plate. The effective thermal diffusivity (α_e) can be written as $\alpha_e=\alpha_0+\alpha_d$, where $\alpha_0=frack\rho C_P$ and $\alpha_d=\gamma^* du$. The effective solutal diffusivity (D_e) can be written as $D_e=D+D_d$ where $D_d=\xi^* du$.