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Convection in a saturated Darcy porous medium with an exothermic chemical surface reaction and Soret effect ☆,☆☆

Nicola L. Scott

Department of Mathematics, University of Durham, DH1 3LE, UK

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ABSTRACT

We consider a saturated Darcy porous medium with an exothermic chemical reaction on the lower boundary. The Dufour effect is neglected, however the influence of the Soret effect is studied. It is found that increasing the Soret coefficient may be either linearly stabilising or destabilising, depending on the boundary parameters and Lewis number. When stationary convection occurs increasing the Soret coefficient has a stabilising effect. A switch from stationary to oscillatory convection occurs at a lower Lewis number for a greater Soret coefficient. Linearised perturbation equations are given and numerical results for the critical Rayleigh number are presented graphically and then discussed.

1. Introduction

In the area of porous media there have been many comprehensive books written including those by Ingham and Pop [4], Nield and Bejan [8], Pop and Ingham [10], Straughan [15], Vadasz [18] and Vafai [19,20]. It is well known in the literature regarding convection in fluids and porous media that a thermal gradient may induce a small matter flow called the Soret effect, this is discussed in, for example, Piazza and Guarino [9] and Soret [14]. This cross-diffusion effect may cause the denser component of a multi-component system to diffuse to a cooler region, considered the positive direction, or to a warmer region, which is the negative direction. When studying many natural processes, such as chemical reactions in sediments, c.f. Domenico [2], the Soret effect may be significant and cannot be neglected. The reciprocal effect in which a solutal gradient causes a thermal flux is called the Dufour effect. In recent years a large number of articles on these cross-diffusion effects have been published and references to these may be found in Section 9.1.4 of Nield and Bejan [8].

Continuous dependence on the Soret coefficient has been established for fixed boundary conditions using both the Brinkman model (Straughan and Hutter [16]) and the Darcy model (Lin and Payne [6]). The influence of both the Soret and Dufour effects has been studied in many different models, vertical studies include those of Tai and Char [17] who considered non-Newtonian fluids and Postelnicu [11] who modelled a chemical reaction in a porous medium, a visco elastic fluid flow over a stretching sheet is investigated by Salem [12]; Alam et al. [1] examine magneto-hydrodynamic mixed convection on an inclined plate and include a chemical reaction and heat generation; Malashetty and Biradar [7] study a Maxwell fluid; and Lakshmi Narayana and Murthy [5] analyse free convection in a horizontal plate of a Darcy porous medium.

In this work we revisit the problem of Scott and Straughan [13], in which a horizontal layer of a Darcy porous medium of depth h bounded on either side by solid walls and saturated by an incompressible fluid is subject to an exothermic reaction on the lower wall, and discuss how the results obtained change with the inclusion of the Soret effect. Maintaining the assumption from Scott and Straughan [13] that the density is independent of the reactant concentration the transport equations for this problem are, c.f. Straughan [15],

$$
\begin{aligned}
&p_{,i} = \frac{\mu}{K} v_i - g\rho_0(1-\alpha(T-T_0))k_i,\\
&v_{i,i} = 0\\
&\frac{1}{M} T_{,t} + v_i T_{,i} = \kappa \Delta T,\\
&\phi C_{,t} + v_i C_{,i} = \phi k_c \Delta C + \phi k_s \Delta T,
\end{aligned}
\tag{1}
$$

where $M = (\rho_0 c_p)_f/(\rho_0 c)_m$, with $(\rho_0 c)_m = \phi(\rho_0 c_p)_f + (1-\phi)(\rho c)_s$, $\kappa = k_m/(\rho_0 c_p)_f$, and $k_m = \kappa_s(1-\phi) + \kappa_f \phi$, where κ_s and κ_f are the thermal conductivities of the solid skeleton of the porous medium and the saturating fluid, respectively. The final term on the right hand side of Eq. $(1)_4$ arises from the inclusion of the Soret effect.

On the upper boundary wall the temperature and concentration are held constant and there is no mass flux across the boundary,

$$
n_i v_i = w = 0, \qquad T = T_U, \qquad C = C_U \qquad \text{on} \quad z = h, \tag{2}
$$

where $\mathbf{n} = (0, 0, 1)$ so that $w \equiv v_3$.

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E-mail address: n.l.scott@durham.ac.uk.

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Nomenclature

A,B,H	dimensionless boundary condition parameters
C	reactant concentration
c	specific heat
c_p	specific heat at constant pressure
E	reaction activation energy
h	depth of porous layer
K	permeability of the porous medium
k	wave number
k_0	reaction rate constant
k_c	diffusivity of the reactant
k_m	thermal conductivity of the porous medium
Le	Lewis number
Q	heat of reaction
R	Rayleigh number
R^*	universal gas constant
S	Soret coefficient
T	temperature
T_0	reference temperature
t	time
$\mathbf{v}$	velocity
α	coefficient of thermal expansion
$\beta_1,\beta_2,\beta_3,\beta_4$	constants determined in system of Eq. (9)
κ	thermal diffusivity of the porous medium
μ	dynamic viscosity
ξ	dimensionless activation energy parameter
ρ	density
ρ_0	reference density
ϕ	porosity

An exothermic surface reaction in which a reactant is converted into an inert product at a rate r, where

$$r=k_0 C\exp\left(\frac{-E}{R^*T}\right),$$

is present on the lower boundary. Here, k_0 is a rate constant, R^* is the universal gas constant and E is the reaction's activation energy.

The heat flux $\mathbf{q}=-k_m\nabla T$ is the rate at which heat crosses the boundary and is obtained from the right hand side of Eq. $(1)_3$. On the lower boundary $\mathbf{q}$ is proportional to the heat of the reaction, Q, and the rate at which it occurs.

The right-hand side of Eq. $(1)_4$ may be written as $\phi\nabla\mathbf{J}$, where $\mathbf{J}=k_c\nabla C+k_s\nabla T$. The flux $\mathbf{J}$ is proportional to the rate at which the reaction occurs and inversely proportional to the porosity.

Finally, we assume that there is no mass flux across the lower wall and obtain the boundary conditions

$$\begin{aligned}&w=0\\ &k_m\frac{dT}{dz}=-Qk_0C\exp\left(\frac{-E}{R^*T}\right),\\ &\phi k_c\frac{dC}{dz}+\phi k_s\frac{dT}{dz}=k_0C\exp\left(\frac{-E}{R^*T}\right)\quad\text{on}\quad z=0.\end{aligned}\tag{3}$$

2. Non-dimensional linear perturbation equations

We follow the standard linear analysis used in Scott and Straughan [13] by introducing small perturbations $(\mathbf{u},\theta,\phi,\pi)$ to the steady state $(\bar{\mathbf{v}},\bar{T},\bar{C},\bar{p})$ and after non-dimensionalising obtain the linear perturbation equations

$$\begin{aligned}0&=\Delta w-R\Delta^*\theta,\\ \theta_{,t}&=-\beta_1 w+\Delta\theta,\\ M\phi\gamma_{,t}&=-\beta_3 w+\frac{1}{Le}\Delta\gamma+S\Delta\theta.\end{aligned}\tag{4}$$

The constants $\beta_1,\beta_2,\beta_3,\beta_4$ are determined by the boundary conditions and calculated at the end of the current section after the perturbations and non-dimensionalisations have been introduced to the boundary conditions, w is now the third component of $\mathbf{u}$, $\Delta^*=\partial^2/\partial x^2+\partial^2/\partial y^2$, $Le=\kappa/(\phi k_c)$ is the Lewis number, the Rayleigh number is defined by

$$R=\frac{\rho_0 g\alpha hKT_U}{\mu\kappa}$$

and the Soret coefficient is

$$S=\frac{\phi k_s}{\kappa}.$$

To find the Rayleigh number for which the system of Eq. (4) first becomes unstable we consider a periodic cell and employ standard Fourier mode analysis, where we let

$$\gamma=\sum_{j=1}^{\infty}e^{\sigma_j t}f_j(x,y)\Phi_j(z),\tag{5}$$

with similar forms to Eq. (5) for w and θ. The function f satisfies $\Delta^* f=-k^2 f$ for a wave number, k, and σ_j is a general eigenvalue.

The real part of σ_j is the growth rate of a perturbation and therefore for the system to be linearly stable we require the real part of all eigenvalues to be negative. We therefore consider a typical eigenvalue, σ, satisfying

$$\begin{aligned}0&=\left(D^2-k^2\right)W+Rk^2\Theta,\\ \sigma\Theta&=-\beta_1 W+\left(D^2-k^2\right)\Theta,\\ M\phi\sigma\Phi&=-\beta_3 W+\frac{1}{Le}\left(D^2-k^2\right)\Phi+S\left(D^2-k^2\right)\Theta,\end{aligned}\tag{6}$$

where $D=d/dz$.

As Φ only appears Eq. $(6)_3$ it may initially seem possible to de-couple this equation, however the reaction boundary conditions mean that this is not possible.

We find the linearised non-dimensional boundary conditions to be

$$W=0,\quad \Theta=0,\quad \Phi=0,\qquad \text{on}\quad z=1\tag{7}$$

and

$$\begin{aligned}&W=0,\\ &D\Theta=-A\Phi,\\ &D\Phi+HD\Theta=B\Phi\quad\text{on}\quad z=0.\end{aligned}\tag{8}$$

where A,B,H,ξ are non-dimensional coefficients defined by

$$A=\frac{Qk_0C_Uh}{T_Uk_m},\qquad B=\frac{k_0h}{\phi k_c},\qquad H=\frac{k_sT_U}{k_cC_U},\qquad \xi=\frac{E}{T_UR}.$$

It now remains for us to calculate the coefficients β_1 and β_3 by evaluating the steady state on the boundary walls. It is found that

$$\begin{aligned}1&=\beta_1+\beta_2,\\ 1&=\beta_3+\beta_4,\\ \beta_1&=-A\beta_4\exp(-\xi/\beta_2),\\ \beta_3+H\beta_1&=B\beta_4\exp(-\xi/\beta_2),\end{aligned}\tag{9}$$