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Generalized flux-gradient technique pairing line-average concentrations on vertically separated paths

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ABSTRACT

Line-averaging optical gas detectors offer new avenues for the indirect estimation of surface/air exchange fluxes. This paper examines an inverse dispersion technique (gFG, for "generalized flux-gradient") that yields an estimate of the gas emission rate Q from surface area sources, based on the difference ΔC between line-averaged mean concentrations along two (or more) paths that are vertically inclined or, if horizontal, are vertically separated. The inversion to extract Q from ΔC can be performed using any satisfactory model of turbulent dispersion over a finite source, motivating the examination here of several analytical solutions to the advection-diffusion equation. Each provides a theoretical value $u_* \Delta C/Q$ for the normalized concentration difference, whence an estimate $\widetilde{Q}$ of the flux can be deduced from measured ΔC and u_* (the latter being the friction velocity, for which any suitable velocity scale could be substituted). Discrepancies between the solutions are explored, and the error that results from wrongly treating the source fetch as infinite is quantified. As the fetch increases, gFG relaxes to the standard flux-gradient technique exploiting the (known) Monin–Obukhov concentration gradient.

1. Introduction

This paper outlines an experimental method for determination of surface-air exchange fluxes "Q" by inverse dispersion, exploiting the flexibility of recently developed line-averaging, open-path optical gas detectors. The technique, which for convenience we label gFG (for "generalized flux-gradient"), is related to the flux-gradient method in that it exploits a vertical difference $(C_1 - C_2)$ in the mean concentration of the gas of interest. However whereas a standard flux-gradient approach derives Q from vertically-separated *point* sensors exposed within a constant flux layer, gFG is based on line-averaged concentrations (along paths, furthermore, that are not necessarily parallel), and it applies even over a limited (but known) fetch of source. Inverse dispersion on the micro-meteorological scale has to date more typically been based on *horizontally*-separated concentration measurements (see survey of Wilson et al., 2012), and in that configuration cannot easily deal with sources of large areal extent or uncertain perimeter.

Suppose the atmospheric surface layer (ASL) is horizontally homogeneous, thus characterized by the friction velocity u_*, Obukhov length L, surface roughness length z_0 and mean wind direction (a single sonic anemometer-thermometer provides information allowing to deduce these quantities). We will align the horizontal coordinate x with the direction of the mean wind and, with the purpose of illustrating gFG in an idealized source geometry, consider a uniform ground-level source of trace gas lying at $x \geq 0$ and extending to infinity along the crosswind (y) axis. Now consider a pair of gas detector paths that originate at $(x_{e1}, 0, z_{e1})$, $(x_{e2}, 0, z_{e2})$, the "emitter" locations, and whose endpoints (the "reflectors") lie respectively at $(x_{e1} + D_{x1}, 0, z_{r1})$ and $(x_{e2} + D_{x2}, 0, z_{r2})$: Fig. 1 shows a case of special interest, where both paths share a common emitter/detector point (x_c, z_c) and lie in the vertical plane at $y = 0$ (this is an eminently practical configuration, as it represents the case of a fixed optical emitter/detector being sequentially aimed to high and low reflectors). We also accommodate the possibility that the emitter point(s) x_e could lie upwind from the leading edge of the source, a configuration that might be chosen if (for example) the source area were a pond, or ground inhospitable to the placement of instruments, or a herd of animals confined within a paddock.

At the end of a measurement interval the instrument provides time-space mean concentrations C_1, C_2 for the lower and upper paths.[1] Assuming that "background" (or upwind) concentration is spatially and temporally uniform on the scale of interest, the

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[1] It would be ideal if the instrument were inherently differential – an ideal that some modern detectors almost (though not quite) attain; in the case of the instrument described in the companion paper (Flesch et al., 2016) *almost*, because the

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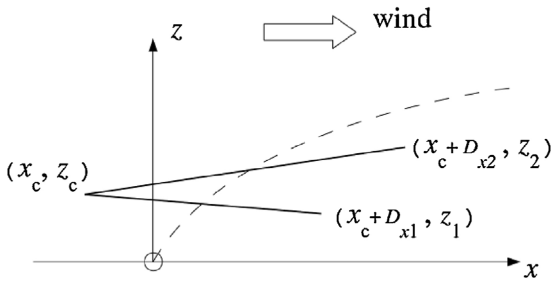

Fig. 1. Schematic of a measurement setup for source estimation by inverse dispersion using a gFG (generalized flux-gradient) method. The upwind edge of the gas source is at the origin ($x = 0$). The equipment returns the time- and line-averaged gas concentrations C_1 and C_2 along respectively the lower and upper grey lines, in this case shown as slant paths with a common emitter/detector position (x_c, z_c). In practice one would prefer that the detection paths lie within the growing gas plume (whose envelope is indicated by the dashed line). For results shown in this paper the two measurement paths have the same projection onto the horizontal axis, i.e. $D_{x1} = D_{x2}$.

difference $\Delta C \equiv (C_1 - C_2)$ is indifferent to its value (C_0). Then if one has a theoretical value $\Phi = u_* \Delta C/Q$ for the "conversion number" relating the unknown source strength Q to the concentration difference, measured u_* and ΔC give an estimate

$$Q_{gFG} = \frac{[u_* \, \Delta C]^{meas}}{\Phi} \tag{1}$$

of the emission rate.

The next section will briefly review available analytical prescriptions for the field of normalized concentration difference $u_* \Delta C/Q$, including solutions of the advection-diffusion equation. In a related paper (Flesch et al., 2016) gFG is performed on the basis of a more advanced Lagrangian stochastic (LS) trajectory model, however the purpose here is to look at the technique more broadly, evaluating a practicable technique that does not carry the computational burden inherent in computing turbulent trajectories: for doing so necessitates a time consuming computation of backward-time trajectories from representative points all along the (slanted) detector paths. In such a context the ~10% level of accuracy is about what one might realistically hope for, certainly for individual inversions (i.e. circa 15–30 min averaging intervals): averaging over repeated trials should narrow that uncertainty.

2. Formulae for the conversion: $u_* \, \Delta C \longrightarrow Q$

Stationarity of both the micrometeorological state and of the tracer field is assumed, and the following notation is used: $\bar{c} = \bar{c}(x, y, z)$ represents the mean concentration at a single point, while C will designate an average value of $\bar{c}$ along a measurement path, i.e C is a shorthand notation for $\langle \bar{c} \rangle$, with $\langle \; \rangle$ designating the line-averaging operation.

2.1. Inversion using the MO concentration profile (infinite fetch implied)

Two estimates of the conversion number $\Phi \equiv u_* \Delta C/Q$, both *neglecting edge effects*, can be extracted from the Monin–Obukhov concentration profile, i.e.

$$\bar{c}^{MO}(z) = \bar{c}^{MO}(z_0) + \frac{C_*}{k_v/S_c} \left[\ln\frac{z}{z_0} - \psi_c\left(\frac{z}{L}\right) + \psi_c\left(\frac{z_0}{L}\right) \right] \tag{2}$$

where C_* ($\equiv -Q/u_*$) is the tracer concentration scale, k_v (= 0.4) is the von Karman constant, and S_c (Schmidt number) is the ratio of the eddy viscosity to the eddy diffusivity in the neutral limit. The diabatic correction function ψ_c is here evaluated as

$$\psi_c = \psi_c(\phi_c) = 2\ln\left[\frac{1}{2}\,\left(1 + \phi_c^{-1}\right)\right], \tag{3}$$

with ϕ_c given by (Dyer and Hicks, 1970)

$$\phi_c = 1 + 5\,z/L, \quad L \geq 0, \tag{4}$$

$$\phi_c = (1 - 16z/L)^{-1/2}, \quad L < 0. \tag{5}$$

Eq. (2) can be applied to compute the difference in the line averaged concentrations ("Φ_{MO}") or the difference between the concentrations at the *midpoints* of the two beams ("Φ_{MO-mid}"): if the MO concentration profile were linear with height, these would coincide. For the MO solutions, the absolute location of the measurement system relative to the edge of the source has no impact on the conversion number $u_* \Delta C/Q$.

2.2. Formulae that account for limited fetch of source

We neglect variation of wind direction with height, and (as already noted) assume sources extend to infinity in the y direction. All existing formulae for surface layer dispersion are solutions, exact or otherwise, of an advection-diffusion equation (ADE), here[2]

$$\bar{u}\;\frac{\partial \bar{c}}{\partial x} = \frac{\partial}{\partial z}\left[K_c\;\frac{\partial \bar{c}}{\partial z}\right]\;, \tag{6}$$

where $\bar{u} = \bar{u}(z)$ is the mean wind profile and $K_c = K_c(z)$ is the profile of the eddy diffusivity for the species "c". The flow being (by assumption) horizontally homogeneous, the proper (Monin–Obukhov) profiles for insertion in Eq. (6) are

$$\bar{u} = \frac{u_*}{k_v}\left[\ln\frac{z}{z_0} - \psi_m\left(\frac{z}{L}\right) + \psi_m\left(\frac{z_0}{L}\right)\right] \tag{7}$$

and

$$K_c = \frac{(k_v/S_c)u_* z}{\phi_c(z/L)}. \tag{8}$$

The MO function ϕ_c for the concentration profile is given above. The corresponding function ϕ_m for the wind profile was specified as (Dyer, 1974; Dyer and Hicks, 1970)

$$\phi_m = 1 + 5\,z/L, \quad L \geq 0, \tag{9}$$

$$\phi_m = (1 - 16\,z/L)^{-1/4}, \quad L < 0, \tag{10}$$

and implies that the diabatic correction function ψ_m in Eq. (7) is

$$\psi_m\left(\frac{z}{L}\right) = -5z/L, \quad L \geq 0, \tag{11}$$

$$\psi_m\left(\frac{z}{L}\right) = 2\ln\left(\frac{1 + \phi_m{}^{-1}}{2}\right) + \ln\left(\frac{1 + \phi_m{}^{-2}}{2}\right) - 2\,\mathrm{atan}\left(\phi_m{}^{-1}\right) + \frac{\pi}{2}, \quad L < 0. \tag{12}$$

Exact solutions of Eq. (6) can be obtained if, in lieu of Eqs. (7) and (8), the profiles of wind speed and diffusivity are parameterized as power laws,

$$\bar{u} = \bar{u}_H(z/H_u)^m = \mathcal{U} z^m, \tag{13}$$

$$K_c = K_{cH}\,(z/H_K)^n = \kappa\, z^n. \tag{14}$$

emitter/detector is common to all paths; but *not quite*, because (for instance) measurements on the paired paths are sequential rather than simultaneous.

[2] Eq. (6) reflects the restrictions of scope outlined above. Neglect of the along wind velocity fluctuation and its correlation with the vertical velocity means that this treatment is less satisfactory for strongly unstable stratification.