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A three dimensional exact equation for the turbulent dissipation rate of Generalised Newtonian Fluids☆

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ABSTRACT

The flow of non-Newtonian fluids is of interest in many biological and industrial applications, including nanofluids. Most of the papers of the literature on turbulent non-Newtonian fluids focused the attention on viscoelastic fluids. In order to make accurate and low cost prediction of turbulent inelastic non-Newtonian fluids, a RANS Generalised Newtonian Fluid (GNF) turbulence model, based on the exact equations for the turbulent variables, is required. In a previous paper of the same authors the exact equations for the turbulent kinetic energy and the dissipation rate have been derived in a two-dimensional (2D) domain, through the introduction of an apparent viscosity equation. The aim of the present paper is to extend the approach to a three-dimensional (3D) domain, giving the full mathematical demonstration of the exact equations.

1. Introduction

The term “non-Newtonian” fluid is very general, including a wide range of fluids with different constitutive equations. The flow of non-Newtonian fluids is largely present in industrial applications and biological problems, as slurries flow in pipes, wastewater treatment and aseptic food processing. Several theoretical solutions and numerical simulations found in the literature are related to laminar flow of non-Newtonian fluids, including two of the first author [1,2]. The most important example of non-Newtonian fluid in biology is blood. Besides that the flow is laminar in most of the vascular network, turbulent conditions may occur in some regions, e.g. bifurcations, producing the formation of atherosclerotic plaques, in association to biochemical factors [3]. Turbulent flow can be encountered in other conditions, like sewage transport, drilling hydraulics and processes with high heat transfer rate. Turbulent flow increases the heat transfer coefficient in the aseptic food processing, due to the large temperature difference, reduces the viscosity and leads the transition from laminar to turbulent flow [4].

The viscoelastic ones are among the most investigated non-Newtonian fluids, because of the drag reduction, called Tom's effect [5]. This feature was studied numerically in a wide number of papers employing different constitutive relations to describe the viscoelastic behaviour of the fluid. The FENE-P model is one of the most popular, and several Direct Numerical Simulations (DNS) were carried out to explain the phenomenon of drag reduction [6]; finding relations between flow and fluid rheological parameters at high, medium and low drag regime [7]; studying the zero-pressure gradient flow in turbulent boundary layers where the polymer is homogeneously distributed in the solvent [8]; and, finally, investigating the influence of the polymers on the turbulence [9]. The Giesekus and the FENE-P models were employed to predict the drag reduction. Examples can be found in [10,11], where simulations with both models were performed, and in [12], where numerical results were compared to Particle Image Velocimetry (PIV) experiments.

DNS is a powerful instrument but its application is limited by computational resources. This is the reason why many DNS simulations are limited to low and medium Reynolds numbers. The models proposed by Poreh and Hassid [13], and Durst and Rastogi [14], were based on the approach of modifying the damping function of the low Reynolds number $k-\varepsilon$ model of Jones and Launder [15]. Others proposals can be found in [16], where a zero-equation model for the eddy viscosity was suggested, or in [17], where closures were developed for turbulent correlations among flow and polymer conformation variables, incorporating a single-point $k-\varepsilon$ model.

A mechanistic model for polymers was suggested in [18], where the dominant forces on a polymer in turbulent flow were argued to be elastic and centrifugal. The corrected velocity profiles, resulting from the dimensional analysis in the turbulent boundary layer, were compared favourably with the experiments of [19]. Due to the complexity of the viscoelastic model, other authors proposed a simpler constitutive equation, inspired by the GNF model. The elongation viscosity was modelled as function of the magnitude of the strain rate and the rotation rate tensors [20], and function of the third and the second invariant of the rate-of-strain tensor [21]. DNS results are provided in order to show the capability of the models to reproduce a

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drag reduction. The role of the stress anisotropy was studied experimentally in [22] with a Laser Doppler Velocimetry (LDV) and numerically by means of DNS. The results of two different constitutive equations were compared showing that the model characterised by a single scalar viscosity function of the second and the third invariant of the rate of shear tensor had a good qualitative agreement with the measurements. An approach similar to that adopted in [21] was followed in [23]. By choosing a Bird–Carreau constitutive equation for the viscosity, depending on the second and the third invariant of the rate of shear tensor, it was showed qualitatively that the introduction of the third invariant of the rate of strain tensor contributed to an increase of the viscous diffusion and of the turbulent dissipation rate. The same constitutive equation was employed in [24], in order to derive a low Reynolds number $k-\varepsilon$ model for drag reducing fluids. An algebraic equation was proposed to correlate the instantaneous viscosity to the dissipation rate while the average viscosity and the dissipation rate were correlated through a normal logarithmic probability distribution. Applying the dimensional analysis it was possible to neglect many terms in the transport equations. The final turbulent dissipation rate equation was written in non-conservative form because the explicit time derivative of the average viscosity was present. The model was completed in [25] on the basis of that proposed in [26]. The values of the parameters and the forms of the damping functions were derived taking into account viscometric and elastic near-wall effects. Simulations of pipe flow viscoelastic polymer solutions were compared to experimental data. The model was improved successively in several other papers. The new stress, i.e. the cross-correlation between the fluctuating viscosity and the fluctuating rate of strain, was added as proportional to the mean velocity gradient [27]. The same model was used modifying the damping functions and the coefficients [28], while the Launder–Sharma model [29] was used in [30], instead of that of Nagano-Hishida [26]. A full Reynolds stress model was developed in [31], comparing the performances of the new model with those of [27] and other experimental data. Removing the dependence of the velocity gradient on the friction velocity in the recirculation zone, the Reynolds stress model performed better than the $k-\varepsilon$ one.

Few numerical investigations dealt with turbulent flow of pseudoplastic (shear-thinning) and dilatant (shear-thickening) fluids because of the lack of models with one or two point closure. Among the investigators who performed DNS, Rudman and Blackburn [32], used the Spectral Element-Fourier Method (SEM) in duct flow, comparing the results of a power law fluid with small consistency index and a Herschel–Bulkley fluid with the experimental data. A turbulent model for a non-Newtonian power law fluid was developed in [33], in analogy to the turbulent viscosity, determining the temperature distribution for soybean milk flowing inside a tubular heat exchanger.

Turbulent flow of non-Newtonian fluids is also important in the medical field. A model was developed in [34] to predict the turbulent flow of a power-law fluid in a bio-reactor for anaerobic digestion with the classical $k-\varepsilon$ model and the power-law viscosity. Equations for $k-\varepsilon$ were derived in [35,36] for a power-law and a Herschel–Bulkley fluid using the apparent viscosity of a non-Newtonian fluid in the RANS equations, showing some agreement with the previous empirical correlations.

Another very important field of application is related to nanofluids, which are dilute liquid suspensions of nanoparticles with at least one of their critical dimensions smaller than about 100 nm [37]. Many experimental studies confirmed that the viscosity of these fluids is temperature and shear rate dependent. In some works the viscosity seems to have a shear-thinning behaviour [38–45], while in others a shear-thickening one [46]. Few numerical simulations were carried out in laminar flow [47–50]. An interesting model was developed in [51], where the effect of the nanoparticle/base-fluid relative velocity is described more mechanistically than in the dispersion models, although the fluid was considered Newtonian.

Besides the many applications of non-Newtonian fluids in turbulent flow, no paper derived exactly the turbulent dissipation rate equation for an inelastic GNF model. In the previous paper of the same authors [52], the exact equation for the turbulent dissipation rate in conservative form was derived in a two-dimensional (2D) domain.

The present work presents the extension to a three-dimensional (3D) domain of the exact conservation equations, assuming viscosity as dependent only on the second invariant of the shear rate, because the third invariant is related to the extensional viscosity, which is not of interest for inelastic non-Newtonian fluids. It can be remarked, though, that this hypothesis may be reasonable for real fluids, as stated in [53], due to some evidence. The equation of the turbulent dissipation rate, ε, is obtained in this work by the equation for the apparent viscosity, introduced in [52] and extended here to a 3D domain, which does not require a constitutive link between apparent viscosity and shear rate, and does not need any hypotheses on the dependence of the turbulent dissipation rate on the fluctuating part of the rate of strain tensor, as required in [14].

The present paper presents the derivation of the equations for the average momentum, the turbulent kinetic energy and then the derivation of the equations for the rate of shear tensor and the shear-rate. The differential equation for the apparent viscosity, which allows deriving the equation for the dissipation rate in conservative form, is obtained with the same approach of [52]. The method used in this work allows classifying each term as transport, production or dissipation one.

2. Conservation equations of mass, momentum and turbulent kinetic energy

The present analysis is carried on for a GNF flow, where the apparent viscosity is function of the shear-rate, $\dot{\gamma}$, only, defined as

$$\dot{\gamma} = \sqrt{2S_{ij}S_{ij}}, \tag{1}$$

being S_{ij} the component of the rate of strain tensor

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}. \tag{2}$$

We will use the following variable S, as shear-rate, in the rest of the paper

$$S = \sqrt{S_{ij}S_{ij}} = \dot{\gamma}/\sqrt{2}. \tag{3}$$

The stress tensor is given by

$$T_{ij} = -p\delta_{ij} + 2\mu_{app}S_{ij}, \tag{4}$$

where p is the static pressure.

The conservation equations of the mean variables are

$$\frac{\partial U_k}{\partial x_k} = 0, \tag{5}$$

for the mass, and

$$\rho\frac{\partial U_i}{\partial t} + \rho U_k\frac{\partial U_i}{\partial x_k} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k}\left(2\overline{\mu_{app}S_{ik}} + T_{ik}^R + T_{ik}^{\mu}\right), \tag{6}$$

for the momentum.

The Reynolds stress tensor is defined as

$$T_{ij}^R = -\overline{\rho u_i' u_j'}, \tag{7}$$