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Free convection in a non-Newtonian fluid along a horizontal plate embedded in porous media with internal heat generation ☆

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ABSTRACT

Similarity solutions for the problem of free convection flow over a non-isothermal horizontal plate embedded in porous media are investigated in the presence of internal heat generation. The porous medium is saturated with non-Newtonian power law fluid. Numerical results are obtained for the effect of power law temperature profile and fluid index on the heat transfer characteristics.

1. Introduction

Nield and Bejan [1], Ingham and Pop [2], Vafai [3] provide an extensive study on the practical importance and applications of thermal convection in porous media. Of late a number of problems in free convection in the presence of internal heat generation source have been investigated [4–9]. The effect of internal heat generation finds its applications in reactor safety analysis, metal waste form development for spent nuclear fuel, fire and combustion studies, and storage of radioactive materials.

Gorla and Kumari [10] give similarity solutions for free convections in non-Newtonian fluids along horizontal plate in the absence of internal heat generation whereas Postelnicu and Pop [8] have studied the problem of free convection over horizontal and vertical surfaces with internal heat generation for Newtonian fluids. In this paper we study the effect of variable temperature profile and fluid index on the velocity and temperature profile in the presence of internal heat generation for a horizontal plate embedded in a porous medium saturated with non-Newtonian fluid. Similarity solutions are obtained for exponentially decaying heat generation term [8] and the resulting system of differential equations is solved numerically.

2. Mathematical formulation

Equations governing the problem of free convection boundary layer from a heated horizontal surface embedded in a saturated porous medium with non-Newtonian fluid are written as [8]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial (u^n)}{\partial y}=-\frac{gK^*(n)\beta}{\nu^*}\frac{\partial T}{\partial x} \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\frac{q_w}{\rho c_{pf}} \qquad (3)$$

where the x – coordinate and the y – coordinate are measured along the plate and normal to the plate. The power law fluid index n for various fluids is as follows:

(i) $n<1$ for pseudo plastic fluids or shear-thinning fluids that have a lower apparent viscosity at higher shear rates.
(ii) $n=1$ for Newtonian fluids where the shear stress is directly proportional to the shear rate.
(iii) $n>1$ for dilatant fluids or shear-thickening fluids for which there is an increase in the apparent viscosity at higher shear rates.

The modified permeability $K^*(n)$ is defined as

$$K^*(n)=\begin{cases}\dfrac{6}{25}\left(\dfrac{n\varepsilon}{3n+1}\right)^n\left(\dfrac{\varepsilon d}{3(1-\varepsilon)}\right)^{(n+1)} & \text{by Christopher and Middleman [11]}\\[2ex] \dfrac{2}{\varepsilon}\left(\dfrac{d\varepsilon^2}{8(1-\varepsilon)}\right)^{(n+1)}\dfrac{(6n+1)}{(10n-3)}\left(\dfrac{16}{75}\right)^{\frac{3(10n-3)}{10n+11}} & \text{by Dharmadhikari and Kale [12]}\end{cases}$$

where d is the diameter of the particle and ε is the porosity of the medium.

The appropriate boundary conditions associated with Eqs. (1)–(3) are

$$v=0,\quad T=T_\infty+Ax^\lambda \qquad \text{at } y=0 \text{ for } x\geq 0 \qquad (4)$$

$$u\to 0,\quad T\to T_\infty \qquad \text{as } y\to\infty \qquad (5)$$

where A and λ are positive constants.

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Nomenclature

A	constant
c_{pf}	specific heat at constant pressure of the fluid
f	dimensionless velocity profile
g	acceleration due to gravity
k_m	effective thermal conductivity
$K^*(n)$	modified permeability of porous medium
Nu_x	local Nusselt number
q_w	internal heat generation
Ra_x	generalized local Rayleigh number
T	temperature
u & v	velocity component in x and y directions
x & y	Cartesian coordinates along to the plate and normal to it respectively

Greek symbols

α_m	effective thermal diffusivity
β	coefficient of thermal expansion
λ	constant
ξ	similarity variable
θ	dimensionless temperature function
ρ	density
ν^*	modified kinematic viscosity
ψ	stream function

Subscripts

w	wall condition
∞	ambient condition

We introduce the similarity transformations

$$\psi = \alpha_m Ra_x^{1/3} f(\xi), \quad \theta(\xi) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \xi = Ra_x^{1/3}\frac{y}{x} \tag{6}$$

where ψ is the stream function defined in the usual way $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. The generalized local Rayleigh number Ra_x is defined as

$$Ra_x = \left(\frac{g\beta K^*(n)(T_w - T_\infty)x^n}{\alpha_m^n \nu^*}\right)^{\frac{3}{2n+1}}$$

The internal heat generation q_w decays exponentially and is given by

$$q_w = \frac{\rho c_{pf}\alpha_m(T_w - T_\infty)}{x^2} Ra_x e^{-\xi}$$

On using Eq. (6), Eqs. (1)–(3) reduce to the system of differential equations

$$n\left(f'\right)^{n-1} f'' + \lambda\theta + \left(\frac{\lambda - n - 1}{2n+1}\right)\xi\theta' = 0 \tag{7}$$

$$\theta'' + \left(\frac{\lambda + n}{2n+1}\right) f\theta' - \lambda\theta f' + e^{-\xi} = 0 \tag{8}$$

Coupled with the boundary conditions

$$\begin{aligned} &f(0) = 0, \quad \theta(0) = 1 \\ &f' \to 0, \quad \theta \to 0 \text{ as } \xi \to \infty \end{aligned} \tag{9}$$

The primes in Eqs. (7)–(9) denote differentiation with respect to the similarity variableξ.

3. Results and discussion

Eqs. (7) and (8) along with the boundary conditions (Eq. (9)) are solved numerically for fluid index n varying from 0.5 to 2.0 for different values of power law index λ. The local Nusselt number is defined by

$$Nu_x = -\frac{x}{T_w - T_\infty}\frac{\partial T}{\partial y}\bigg|_{y=0} = -Ra_x^{1/3}\theta'(0)$$

Table 1
Values of $Nu_x/Ra^{-1/3}$ for different values of n and λ.

n	$\lambda = 0$		$\lambda = 1/4$		$\lambda = 1/2$		$\lambda = 1$	
	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation
0.5	−0.1568	0.4259	0.2554	0.6760	0.5264	0.8720	0.9611	1.1443
0.8	−0.1919	0.4249	0.1691	0.6500	0.4261	0.8214	0.8355	1.1075
1.0	−0.1992	0.4240	0.1436	0.6469	0.3943	0.8163	0.7930	1.0994
1.5	−0.2005	0.4460	0.1163	0.6494	0.3563	0.8171	0.7370	1.1033
2.0	−0.2051	0.4606	0.1076	0.6607	0.3407	0.8240	0.7099	1.1131

Table 2
Velocity f' at the leading edge $\xi = 0$ for different values of n and λ.

n	$\lambda = 0$		$\lambda = 1/4$		$\lambda = 1/2$		$\lambda = 1$	
	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation	With internal heat generation	Without internal heat generation
0.5	2.3657	1.5643	2.1643	1.4669	2.1901	1.5448	2.3819	1.6686
0.8	1.6116	1.1558	1.5943	1.1679	1.6419	1.2192	1.7747	1.3438
1.0	1.4133	1.0476	1.4309	1.0885	1.4809	1.1408	1.5947	1.2512
1.5	1.1961	0.9533	1.2396	1.0061	1.2870	1.0536	1.3735	1.1533
2.0	1.1058	0.9279	1.1579	0.9788	1.2005	1.0234	1.2715	1.1170