International Communications in Heat and Mass Transfer 37 (2010) 1064–1071

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet☆

Bikash Sahoo [1], Younghae Do *

Department of Mathematics, Kyungpook National University, Daegu, 702-701, Republic of Korea

ARTICLE INFO

Available online 11 August 2010

Keywords:
Third grade fluid
Partial slip
Magnetic field
Heat transfer
Finite difference method
Broyden's method

ABSTRACT

The entrained flow and heat transfer of an electrically conducting non-Newtonian fluid due to a stretching surface subject to partial slip is considered. The partial slip is controlled by a dimensionless slip factor, which varies between zero (total adhesion) and infinity (full slip). The constitutive equation of the non-Newtonian fluid is modeled by that for a third grade fluid. The heat transfer analysis has been carried out for two heating processes, namely, (i) with prescribed surface temperature (PST case) and (ii) prescribed surface heat flux (PHF case). Suitable similarity transformations are used to reduce the resulting highly nonlinear partial differential equations into ordinary differential equations. The issue of paucity of boundary conditions is addressed and an effective second order numerical scheme has been adopted to solve the obtained differential equations. The important finding in this communication is the combined effects of the partial slip, magnetic field and the third grade fluid parameter on the velocity, skin-friction coefficient and the temperature field. It is interesting to find that slip decreases the momentum boundary layer thickness and increases the thermal boundary layer thickness, whereas the third grade fluid parameter has an opposite effect on the thermal and velocity boundary layers.

1. Introduction

The analysis of boundary layer flows of viscous fluids driven by a moving surface dates back to the pioneering investigation by Sakiadis [1]. There are situations when the extruded polymer sheet is being stretched as it is being extruded and this is bound to alter significantly the boundary layer characteristics of the flow considered by Sakiadis [1]. Both the flow and heat transfer in a viscous fluid over a stretching surface have been extensively investigated during the past decades owing to its importance in industrial and engineering applications. The steady two-dimensional laminar flow of an incompressible, viscous fluid past a stretching sheet has been studied by [2–4].

The aforementioned investigations were restricted to the flows of Newtonian fluids. A rigorous study of the boundary layer flow and heat transfer of different non-Newtonian fluids past a stretching sheet was required due to its immense industrial applications. Rajagopal et al. [5] have considered the flow of a viscoelastic second order fluid past a stretching sheet and obtained the numerical solution of the fourth order nonlinear differential equation. Andersson [6] and Ariel [7] have reported the analytical closed form solutions of the fourth order nonlinear differential equations arising due to the MHD flow and heat transfer of viscoelatic Walters' B' fluid and the second grade fluid respectively. One can further refer the work of Liu [8] and all the references therein regarding the flow and heat transfer of viscoelastic second grade fluid with diverse physical effects. Sahoo and Sharma [9] have carried out an analysis to study the existence, uniqueness and behavior of the fourth order nonlinear coupled ordinary differential equations arising due to the flow and heat transfer of an electrically conducting second grade fluid past a stretching sheet. Although extensive existing investigation of second grade fluid model exhibit normal stresses but for steady flow it does not describe the property of shear thinning or thickening. The non-Newtonian power-law fluid, the modified second grade fluid [10] and the higher grade fluids of differential type [11,12], namely, the third grade and the fourth grade fluids exhibit the shear thinning and shear thickening properties. Recently, Kiwan and Ali [22] have discussed the flow and heat transfer characteristics over a linearly stretching surface with suction or injection in the presence of porous media and internal heat generation or absorption for a uniform temperature.

We now confine to the flow of third grade fluid, driven by a planar stretching surface. The flow is governed by a highly nonlinear boundary value problem in which the order of the differential equation is one more than the number of available boundary conditions. Most recently, the present author [13] has adopted an effective numerical scheme to solve the resulting system of highly nonlinear differential equations arising due to the boundary layer flow and heat transfer of a third grade fluid past a stretching sheet. To the best of the author's knowledge, no attention has been given to the effects of partial slip on the MHD flow and heat transfer of a third grade fluid past a stretching sheet. The objective of the present study

☆ Communicated by W.J. Minkowycz.

* Corresponding author.
E-mail address: yhdo@knu.ac.kr (Y. Do).
[1] On leave from National Institute of Technology, Rourkela, Rourkela-769008, India.

0735-1933/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2010.06.018

is to investigate the combined effects of the non-Newtonian flow parameters, magnetic field and the partial slip on the flow and heat transfer of an electrically conducting third grade fluid arising due to the linearly stretching sheet. The obtained results have promising applications in engineering. The current investigation is not only important because of its technological significance, but also in view of the interesting mathematical features presented by the equations governing the slip flow and heat transfer.

2. Formulation of the problem

It is well known that the Cauchy stress for an incompressible homogeneous third grade fluid is given by [14]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_3(tr\,\mathbf{A}_1^2)\mathbf{A}_1 \tag{1}$$

We consider the steady, laminar flow and heat transfer of an electrically conducting, incompressible and thermodynamically compatible third grade fluid past stretching sheet. The sheet (see Fig. 1) coincides with the plane $y=0$. By applying two equal and opposite forces along the x-axis, the sheet is being stretched with a speed proportional to the distance from the fixed origin, $x=0$. The fluid occupies the half space $y>0$ and the motion of the otherwise quiescent fluid is induced due to the stretching of the sheet. A transverse uniform magnetic field $\mathbf{B}=(0,B_0,0)$ is applied at the surface of the sheet. The fluid adheres to the sheet partially and thus, motion of the fluid exhibits the slip condition. The heat transfer analysis has been carried out for two heating processes, namely, the (i) Prescribed Surface Temperature case (PST) and (ii) Prescribed Heat Flux case (PHF).

2.1. Flow analysis

Making the usual boundary layer approximations [15–17] for the non-Newtonian third grade fluid, namely, within the boundary layer $u, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial p}{\partial x}$ are $O(1)$, y and v are $O(\delta)$, ν and $\frac{\alpha_i}{\rho}(i=1,2)$ being $O(\delta^2)$, β_3 being $O(\delta^4)$ and the terms of $O(\delta)$ are neglected (δ being the boundary layer thickness), the equations of continuity and motion can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\left[u\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] + \frac{2\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + \frac{6\beta_3}{\rho}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u. \tag{3}$$

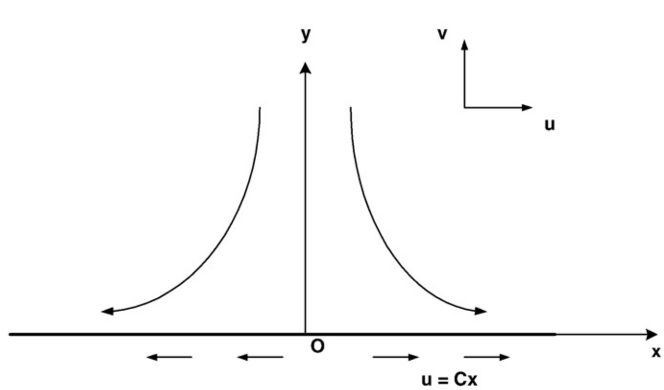

Fig. 1. Schematic diagram of the flow domain.

The appropriate Navier's boundary conditions for the velocity field are

$$u - Cx = \lambda_1\left[\frac{\partial u}{\partial y} + \frac{\alpha_1}{\mu}\left(2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + v\frac{\partial^2 u}{\partial y^2} + u\frac{\partial^2 u}{\partial x\partial y}\right) + 2\frac{\beta_3}{\mu}\left(\frac{\partial u}{\partial y}\right)^3\right] \text{ at } y=0,$$
$$v(0)=0, \quad u\to 0 \quad \text{as} \quad y\to\infty. \tag{4}$$

For similarity solution, we define the variables,

$$u = Cx\varphi'(\zeta), \quad v = -\sqrt{\frac{C\mu}{\rho}}\varphi(\zeta), \quad \text{where} \quad \zeta = \sqrt{\frac{C\rho}{\mu}}y. \tag{5}$$

The continuity Eq. (2) is automatically satisfied. Equations of motion Eq. (3) and the boundary conditions Eq. (4) get reduced to

$$\varphi''' - \varphi'^2 + \varphi\varphi'' + K(2\varphi'\varphi''' - \varphi\varphi^{iv}) - (3K+2L)\varphi''^2 + 6\beta R_x\varphi'''\varphi''^2 - M_n\varphi' = 0, \tag{6}$$

and

$$\varphi(0)=0, \quad \varphi'(0)-1 = \lambda\varphi''(0)[1 + 3K\varphi'(0) + 2\beta R_x\varphi''^2(0)],$$
$$\varphi'(\zeta)\to 0 \quad \text{as} \quad \zeta\to\infty. \tag{7}$$

where $K=\frac{C\alpha_1}{\mu}$, $L=\frac{C\alpha_2}{\mu}$, $\beta=\frac{C^2\beta_3}{\mu}$, $R_x=\frac{Cx^2}{\nu}$ and $M_n=\frac{\sigma B_0^2}{\rho C}$ are respectively the non-dimensional viscoelastic parameter, cross-viscous parameter, the third grade fluid parameter, the local Reynolds number and the magnetic parameter respectively. The relative importance of the slip to viscous effects is indicated by the non-dimensional slip factor $\lambda = \lambda_1\sqrt{\frac{C}{\nu}}$.

Another quantity of interest in the boundary layer flow is the local skin-friction coefficient or frictional drag coefficient, which is related to the wall shear stress $T_{xy}|_{y=0}$, and is given by

$$C_f(x) = \frac{T_{xy}|_{y=0}}{\frac{1}{2}\rho(Cx)^2}, \tag{8}$$

which in terms of the dimensionless quantities is

$$C_f(x) = \frac{2}{\sqrt{R_x}}[\varphi'' + K(3\varphi'\varphi'' - \varphi\varphi''') + 2\beta R_x\varphi''^3]|_{\zeta=0}. \tag{9}$$

2.2. Heat transfer analysis

The thermal boundary layer equation for the thermodynamically compatible third grade fluid with viscous dissipation, work done due to deformation and Joule heating is

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \alpha_1\frac{\partial u}{\partial y}\frac{\partial}{\partial y}\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}\right) + 2\beta_3\left(\frac{\partial u}{\partial y}\right)^4 + \sigma B_0^2u^2. \tag{10}$$

The solution of Eq. (10) depends on the nature of the prescribed boundary conditions. Two types of heating processes are considered as discussed below.

2.2.1. The prescribed surface temperature (PST case)

In this case the boundary conditions are

$$T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 \quad \text{at} \quad y=0,$$
$$T\to T_\infty \quad \text{as} \quad y\to\infty. \tag{11}$$