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Slip flow and heat transfer in axially moving micro-concentric cylinders ☆

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ABSTRACT

The hydrodynamics and thermal behaviors of fluid flow in axially moving micro-concentric cylinders are investigated analytically. Effects of Knudsen number, velocity and radius of the cylinders on the microchannel hydrodynamics and thermal behaviors are investigated. It is found that as Kn increases the slip in the hydrodynamic and thermal boundary condition increases. The slip and the jump at the inner surface are much larger than that of the outer one. When the outer cylinder velocity approaches the inner cylinder one, the slip velocity vanishes. Also, the effect of the variation of U_1 on the temperature jump for adiabatic outer surface is insignificant.

1. Introduction

Fluid flow in microchannels has emerged as an important area of research. This has been motivated by their various applications such as medical and biomedical use, computer chips, and chemical separations. The advent of micro-electro-mechanical systems (MEMS) has opened up a new research area where non-continuum behavior is important. MEMS are one of the major advances of industrial technologies in the past decades. MEMS refer to devices which have a characteristic length of less than 1 mm but greater than 1 μm, which combine electrical and mechanical components and which are fabricated using integrated circuit fabrication technologies. Micronsize mechanical and biochemical devices are becoming more prevalent both in commercial applications and in scientific research. Microchannels are the fundamental part of microfluidic systems. In addition to connecting different devices, microchannels are also utilized as biochemical reaction chambers, in physical particle separation, in inkjet print heads, in infrared detectors, in diode lasers, in miniature gas chromatographs, or as heat exchangers for cooling computer chips. Understanding the flow characteristics of microchannel flows is very important in determining pressure distribution, heat transfer, and transport properties of the flow. The characteristic dimension associated with the term “microchannels” is ambiguous. Nominally, microchannels may be defined as channels whose characteristic dimensions are from one micron to one millimeter. Typical applications may involve characteristic dimensions in the range of approximately 10 to 200 μm. Generally, above one millimeter the flow exhibits behavior which is the same as continuum flows. The Knudsen number (Kn) relates the molecular mean free path of gas to a characteristic dimension of the duct. Knudsen number is very small for continuum flows. However, for microscale gas flows where the gas mean free path becomes comparable with the characteristic dimension of the duct, the Knudsen number may be greater than 10^{-3}. Microchannels with characteristic lengths on the order of 100 μm would produce flows inside the slip regime for gas with a typical mean free path of approximately 100 nm at standard conditions. The slip flow regime to be studied here is classified as $10^{-3}<Kn<10^{-1}$. When the molecular mean free path is comparable to the channel's characteristic dimension, the continuum assumption is no longer valid and the gas exhibits non-continuum effects such as velocity slip and temperature jump at the channel walls. Traditional examples of non-continuum gas flows in channels include low-density applications such as high-altitude aircraft or vacuum technology. The recent development of microscale fluid systems has motivated great interest in this field of study. Microfluidic systems must take into account noncontinuum effects. There is strong evidence to support the use of Navier–Stokes and energy equations to model the slip flow problem, while the boundary conditions are modified by including velocity slip and temperature jump at the channel walls [1–4]. The small length scales commonly encountered in microfluidic devices suggest that rarefaction effects are important. For example, experiments conducted by Arkilic et al. [2,3], Liu et al. [4], Pfalher et al. [5,6], Harley et al. [7], Choi et al. [8], and Wu et al. [9]. The flows studied by Arkilic et al. [2,3] are mostly within the slip flow regime, only bordering the transition regime near the outlet. When using the Navier–Stokes equations with slip flow boundary conditions, the model was able to predict the flow accurately. Liu et al. [4] has also proven that the solution to the Navier–Stokes equation combined with slip flow boundary conditions show good agreement with the experimental data in microchannel flows. The analytical study of internal flows with slip previously has been confined to simple geometries.

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Nomenclature

Kn	Knudsen number, λ/r_o
k	thermal conductivity, $[\mathrm{W\cdot m^{-1}\cdot K^{-1}}]$
Pr	Prandtl number, ν/α
q	conduction heat flux, $[\mathrm{W\cdot m^{-2}}]$
q_o	reference conduction heat flux, $k\Delta T/L$
Q	dimensionless conduction heat flux, q/q_o
$\bar{r}$	radial coordinate, [m]
$\bar{r}_o$	radius of the inner cylinder, [m]
$\bar{r}_1$	radius of the outer cylinder, [m]
r	dimensionless radius, $\bar{r}/\bar{r}_1$
R	dimensionless outer radius, $\bar{r}_1/\bar{r}_o$
T	temperature, [K]
T_o	temperature of the inner cylinder, [K]
T_1	temperature of the outer cylinder, [K]
$\bar{u}$	tangential velocity, [m/s]
$\bar{u}_o$	inner cylinder velocity, [m/s]
$\bar{u}_1$	outer cylinder velocity, [m/s] U dimensionless velocity, $\bar{u}/\bar{u}_0$
U_o	dimensionless inner cylinder velocity, $\bar{u}_o/\bar{u}_0=1$
U_1	dimensionless outer cylinder velocity, $\bar{u}_1/\bar{u}_0$

Greek symbols

α	thermal diffusivity, $[\mathrm{m^2\cdot s^{-1}}]$
ΔT	temperature difference, T_1-T_o
$\Delta\phi$	dimensionless temperature difference
Ω	$=\dfrac{2-\sigma_T}{\sigma_T}\left(\dfrac{2\gamma}{\gamma+1}\right)$
γ	specific heat ratio
λ	mean free path, [m]
δ	$=\dfrac{2-\sigma_v}{\sigma_v}$
ν:	Kinematic viscosity $[\mathrm{m^2\ s^{-1}}]$.
ρ:	Density $[\mathrm{kg\ m^{-3}}]$.
σ_T	thermal accommodation coefficient

Gas flows in microchannels have received considerable attention and have been experimentally, numerically and analytically studied by many researchers [10–12].

The analyses of laminar heat transfer in slip flow regime were first undertaken by Sparrow et al. [13] and Inman [14] for tubes with uniform heat flux and a parallel plate channel or a circular tube with uniform wall temperature using continuum theory subject to slip velocity and temperature jump boundary conditions. The slip flow problem in microtubes has been widely conducted by investigators [15–18]. However, slip flow in microchannels has not been conducted as much as the slip flow in microtubes. Since the slip flow in microchannels requires a two-dimensional approach, its solution is relatively difficult compared to that of the slip flow in microtubes. Some of the slip flow studies in microchannels are summarized here.

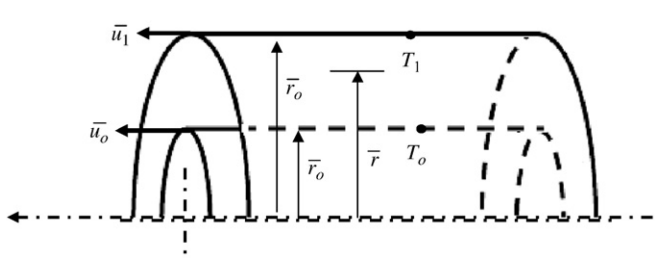

Fig. 1. Schematic diagram of the problem under consideration.

Yu and Ameel [19] studied slip flow heat transfer in microchannels and found that heat transfer increases, decreases, or remains unchanged, compared to non-slip flow conditions, depending on two dimensionless variables that include effects of rarefaction and the fluid/wall interaction. Then, there has been an enormous interest in research works of micro-system over the last decade [20–40].

2. Mathematical model

Consider two long concentric cylinders with a viscous fluid between them, as shown in Fig. 1. The inner cylinder has radius $\bar{r}_o$ and moves axially at $\bar{u}_o$, and the temperature T_o, while the outer cylinder has $\bar{r}_1$,$\bar{u}_1$, and T_1, respectively. The geometry is such that the only nonzero velocity component is u_z, therefore, the variables u_z, and T must be functions only of $\bar{r}$. The fluid is assumed to be Newtonian with uniform properties. Also, it is assumed that the internal heat generation is absent. Referring to Fig. 1 and using the dimensionless parameters given in the nomenclature, the governing equations of the hydrodynamic and thermal behaviors are given by:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dU}{dr}\right)=0, \tag{1a}$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\phi}{dr}\right)+\left(\frac{\mu u_o^2}{k\Delta T}\right)\left(\frac{dU}{dr}\right)^2=0, \tag{1b}$$

Eq. (1a–1b) assume the following boundary conditions:

$$U(1)-1=\delta Kn\frac{\partial U(r)}{\partial r}\bigg|_{r=1},\quad \phi(1)=Kn\frac{\Omega}{\mathrm{Pr}}Q(r)\bigg|_{r=1}, \tag{2a, b}$$

$$U(R)-U_1=-\delta Kn\frac{\partial U(r)}{\partial r}\bigg|_{r=R},\quad \phi(R)-1=-Kn\frac{\Omega}{\mathrm{Pr}}Q(r)\bigg|_{r=R}, \tag{2c, d}$$

where $\delta=(2-\sigma_v)/\sigma_v$, $\Omega=[(2-\sigma_T)/\sigma_T][2\gamma/(\gamma+1)]$ and $Kn=\lambda/r_o$, (Knudson number). Also, the dimensionless slip flow and temperature jump boundary conditions are given respectively, as,

$$\Delta U|_{r=1}=U(1)-1=\delta Kn\frac{\partial U(r)}{\partial r}\bigg|_{r=1},\quad \Delta\phi|_{r=1}=\phi(1)=Kn\frac{\Omega}{\mathrm{Pr}}Q|_{r=1} \tag{3a, b}$$

$$\Delta U|_{r=R}=U(R)-U_1=-\delta Kn\frac{\partial U}{\partial r}\bigg|_{r=R},\quad \Delta\phi|_{r=R}=\phi(R)-1=-Kn\frac{\Omega}{\mathrm{Pr}}Q|_{r=R} \tag{3c, d}$$

where the right hand side of Eq. (3a,b) and (3c,d) represents the slip

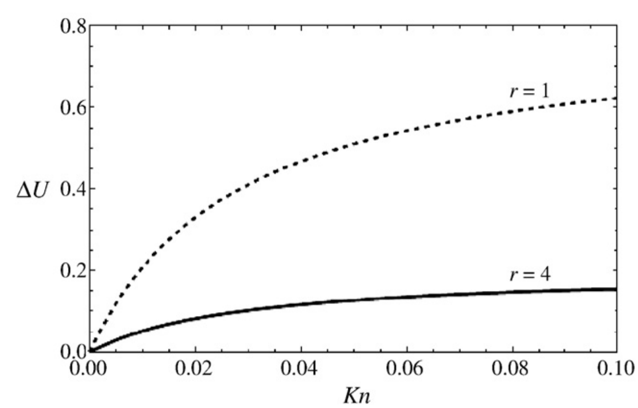

Fig. 2. Effect of Kn on the velocity difference at the walls. $U_1=0$.