International Communications in Heat and Mass Transfer 39 (2012) 72–77

Contents lists available at SciVerse ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Soret and Dufour effects on free convection heat and mass transfer from an arbitrarily inclined plate in a porous medium with constant wall temperature and concentration ☆

Ching-Yang Cheng

Department of Mechanical Engineering, Southern Taiwan University, Yungkang 71005, Taiwan

ARTICLE INFO

Available online 23 September 2011

Keywords:
Arbitrarily inclined plate
Porous medium
Free convection
Soret effect
Dufour effect

ABSTRACT

The free convection boundary layer flow over an arbitrarily inclined heated plate in a porous medium with Soret and Dufour effects is studied by transforming the governing equations into a universal form. The generalized equations can be used to derive the similarity solutions for limiting cases of horizontal and vertical plates and to calculate the heat and mass transfer characteristics between these two limiting cases. The heat and mass transfer characteristics are presented as functions of Soret parameter, Dufour parameter, inclination variable, Lewis number, and buoyancy ratio. Results show that an increase in the Dufour parameter tends to decrease the local heat transfer rate, and an increase in the Soret parameter tends to decrease the local mass transfer rate. As the inclination variable increases, the local Nusselt number and the local Sherwood number decrease from their respective values for horizontal plates, reach their respective minima, and then increase to their respective values for vertical plates. The minima are where the tangential and normal components of buoyancy force are comparable.

1. Introduction

Natural convection heat and mass transfer in a porous medium saturated with Newtonian fluids may be met in geophysical, geothermal and industrial applications, such as the migration of moisture through air contained in fibrous insulations and the underground spreading of chemical contaminants through water-saturated soil.

Cheng and Chang [1] studied the natural convection heat transfer from impermeable horizontal surfaces in a saturated porous medium. Bejan and Khair [2] examined the natural convection boundary layer flow driven by both temperature and concentration gradients. Lai and Kulacki [3] studied the natural convection boundary layer along a vertical surface with constant heat and mass flux. Nakayama and Hossain [4] presented an integral treatment for combined heat and mass transfer by natural convection in a porous medium. Pop and Na [5] studied the natural convection heat transfer from an arbitrarily inclined plate in a porous medium. Li and Lai [6] studied the double diffusive natural convection from horizontal surfaces in porous media.

Cheng [7] examined the effect of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media by an integral approach. Yih [8] studied the uniform transpiration effect on coupled heat and mass transfer in mixed convection about inclined surfaces in porous media for the entire regime. Yih [9] studied the heat and mass transfer driven by natural convection from a truncated cone embedded in a porous medium with variable wall temperature and concentration or variable heat and mass flux. Chamkha and Khaled [10] examined the hydromagnetic simultaneous heat and mass transfer by mixed convection from a vertical plate embedded in a uniform porous medium. Murthy and Singh [11] studied the heat and mass transfer by natural convection near a vertical surface embedded in a non-Darcy porous medium. Cheng [12] presented an integral approach for heat and mass transfer by natural convection from truncated cones in porous media with variable wall temperature and concentration. Cheng [13] examined the double diffusive natural convection along an inclined wavy surface in a porous medium.

Soret effect referred to species differentiation developing in an initial homogeneous mixture submitted to a thermal gradient. The Dufour effect referred to heat flux produced by a concentration gradient. Postelnicu [14] examined the heat and mass characteristics of free convection about a vertical surface embedded in a saturated porous medium subjected to a magnetic field by considering the Dufour and Soret effects. Partha et al. [15] studied the Soret and Dufour effects in a non-Darcy porous medium. Lakshmi Narayana et al. [16] examined the Soret and Dufour effects in a doubly stratified Darcy porous medium. Lakshmi Narayana and Murthy [17] studied the Soret and Dufour effects on free convection heat and mass transfer from a horizontal flat plate in a Darcy porous medium. Mahdy [18] examined the problem of MHD non-Darcian free convection from a vertical wavy surface embedded in porous media in the presence of

☆ Communicated by W.J. Minkowycz.

E-mail address: cycheng@mail.stut.edu.tw.

0735-1933/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2011.09.003

Nomenclature

A	inclination angle
C	concentration
D	Dufour parameter
$\bar{D}$	Dufour coefficient
D_M	mass diffusivity of the porous medium
g	gravitational acceleration
f	dimensionless stream function
K	permeability of porous medium
Le	Lewis number
N	buoyancy ratio
Nu	local Nusselt number
Ra	modified Rayleigh number
S	Soret parameter
$\bar{S}$	Soret coefficient
Sh	local Sherwood number
T	temperature
$\bar{u}, \bar{v}$	velocity components
$\bar{x}, \bar{y}$	Cartesian coordinates

Greek symbols

α	thermal diffusivity of the porous medium
β_T	coefficient of concentration expansion
β_C	coefficient of thermal expansion
η, ξ	dimensionless coordinates
θ	dimensionless temperature
ν	kinematic viscosity
ϕ	dimensionless concentration
ψ	stream function

Subscripts

w	condition at wall
∞	condition at infinity

Soret and Dufour effect. Cheng [19] studied the Soret and Dufour effects on free convection boundary layer over a vertical cylinder in a saturated porous medium. Cheng [20] examined the Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration.

This work aims to study the Soret and Dufour effects on free convection heat and mass transfer above an arbitrarily inclined plate in a porous medium. The surface of the inclined plate is kept at constant temperature and concentration. The governing equations are transformed into a set of coupled differential equations, and the obtained boundary layer equations are solved by the cubic spline collocation method [21]. The effects of Soret parameter, Dufour parameter, inclination, Lewis number, and buoyancy ratio on the heat and mass transfer characteristics over an arbitrarily inclined plate in a porous medium saturated with a Newtonian fluid are carefully examined.

2. Analysis

Consider the boundary layer flow driven by natural convection with temperature and concentration gradients above a semi-infinite plate embedded in a porous medium saturated with a Newtonian fluid in the presence of Soret and Dufour effects. This plate is above the horizontal and is inclined at an angle A ($0^\circ \le A \le 90^\circ$) to the horizontal. The surface of the inclined plate is maintained at a constant temperature T_w greater than the porous medium temperature T_∞ sufficiently far from the inclined plate. The concentration of a certain constituent in the solution that saturates the porous medium varies from a higher concentration C_w on the fluid side of the surface of the inclined plate to a lower concentration C_∞ sufficiently far from the inclined plate.

Based on the boundary layer and Boussinesq approximations, we can write the governing equations for boundary layer Darcy flow by free convection of a Newtonian fluid embedded in a porous medium near an arbitrarily inclined plate with Soret and Dufour effects in two-dimensional Cartesian coordinates ($\bar{x}$, $\bar{y}$) as [5, 22]

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{v}}{\partial \bar{x}} = \frac{gK}{\nu}\left[\beta_T \frac{\partial T}{\partial \bar{y}} \sin A + \beta_C \frac{\partial C}{\partial \bar{y}} \sin A - \beta_T \frac{\partial T}{\partial \bar{x}} \cos A - \beta_C \frac{\partial C}{\partial \bar{x}} \cos A\right] \tag{2}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \alpha \frac{\partial^2 T}{\partial \bar{y}^2} + \bar{D}\frac{\partial^2 C}{\partial \bar{y}^2} \tag{3}$$

$$\bar{u}\frac{\partial C}{\partial \bar{x}} + \bar{v}\frac{\partial C}{\partial \bar{y}} = D_M \frac{\partial^2 C}{\partial \bar{y}^2} + \bar{S}\frac{\partial^2 T}{\partial \bar{y}^2}. \tag{4}$$

The boundary conditions are written as

$$T = T_w, \quad C = C_w, \quad \bar{v} = 0 \quad \text{on} \quad \bar{y} = 0 \tag{5}$$

$$C \to C_\infty, \quad T \to T_\infty, \quad \bar{u} \to 0 \quad \text{as} \quad \bar{y} \to \infty \tag{6}$$

Here $\bar{u}$ and $\bar{v}$ are the volume-averaged velocity components in the $\bar{x}$ and $\bar{y}$ directions, respectively. T and C are the volume-averaged temperature and concentration, respectively. β_T and β_C are the coefficients for thermal expansion and for concentration expansion of the saturated porous medium, respectively. ν and K are the kinematic viscosity of the fluid and the permeability of the porous medium, respectively. α and D_M are the thermal diffusivity and mass diffusivity of the porous medium, respectively. $\bar{D}$ and $\bar{S}$ are the Dufour coefficient and Soret coefficient of the porous medium, respectively. g is the gravitational acceleration.

Here we introduce a nondimensional parameter

$$R = \frac{(Ra \sin A)^{1/2}}{(Ra \cos A)^{1/3}} \tag{7}$$

where $Ra = gK\beta_T(T_w - T_\infty)\,\bar{x}/(\alpha\nu)$ is the Rayleigh number. This parameter represents the relative strength of the longitudinal to normal components of the buoyancy force within the boundary layer.

Here we define the nondimensional variables:

$$\theta = (T - T_\infty)/(T_w - T_\infty) \quad , \qquad \phi = (C - C_\infty)/(C_w - C_\infty) \qquad \xi = R/(1+R) \quad ,$$

$$\eta = (\bar{y}/\bar{x})\left[(Ra \cos A)^{1/3} + (Ra \sin A)^{1/2}\right], \quad f = \left(\bar{\psi}/\alpha\right)\left[(Ra \cos A)^{1/3} + (Ra \sin A)^{1/2}\right]^{-1}. \tag{8}$$

Here we introduce the stream function $\bar{\psi}$ to satisfy the relations:

$$\bar{u} = \partial \bar{\psi}/\partial \bar{y}, \; \bar{v} = -\partial \bar{\psi}/\partial \bar{x}. \tag{9}$$

Substituting Eqs. (7)–(9) into Eqs. (1)–(4), we obtain the following equations:

$$f'' - \frac{1}{6}(4-\xi)(1-\xi)^3 \eta \left(\theta' + N\phi'\right) = \xi^2\left(\theta' + N\phi'\right) - \frac{1}{6}\xi(1-\xi)^4\left(\frac{\partial \theta}{\partial \xi} + N\frac{\partial \phi}{\partial \xi}\right) \tag{10}$$