International Communications in Heat and Mass Transfer 39 (2012) 209–215

Contents lists available at SciVerse ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Numerical predictions of natural convection with liquid fluids contained in an inclined arc-shaped enclosure ☆

Chin-Lung Chen [a,*], Chin-Hsiang Cheng [b]

[a] Department of Mechanical Engineering, Lee-Ming Institute of Technology, New Taipei, Taiwan, ROC
[b] Department of Aeronautics and Astronautics, National Cheng Kung University, Tainan, Taiwan, ROC

ARTICLE INFO

Available online 18 November 2011

Keywords:
Natural convection
Inclined
Arc-shaped enclosure

ABSTRACT

In the present paper a numerical study has been performed of the flow behavior and natural convection heat transfer characteristics of liquid fluids contained in an inclined arc-shaped enclosure. The governing equations are discretized using the finite-volume method and curvilinear coordinates. The Prandtl number (Pr) of the liquid fluids is assigned to be 4.0 and the Grashof number (Gr) is ranged within the regime $1\times10^5 \leqq \leqq 4\times10^6$. On the other hand, the inclination angle (θ) of the enclosure is varied within $0^\circ \leqq \theta \leqq 360^\circ$. Of major concern are the effects of the inclination and the buoyancy force on the flow and the thermal fields, and based on the numerical data of the thermal field the local and overall Nusselt numbers are calculated. Results show that the arc-shaped enclosure for Pr = 4.0 at $Gr = 4\times10^6$ and $\theta = 90^\circ$ exhibits the best heat transfer performance. The poor heat transfer performance for Pr = 4.0 fixed at $Gr = 1\times10^5$ and $\theta = 180^\circ$ exhibits the arc-shaped enclosure, respectively. As the value of Grashof number is elevated from 10^5 to 4×10^6, at $\theta = 90^\circ$, the magnitude of Nu is elevated from 13.946 to 25.3 (81.4% increase); however, at $\theta = 180^\circ$, the magnitude is elevated from 11.655 to 13.475 (15.6% increase) only.

1. Introduction

In the past several decades, natural convection heat transfer in an enclosure with complex shape has been an interesting topic relevant to the design of a large number of thermal devices such as concentrating solar collectors [1–3], lubrication systems [4–5], and electronic cooling units [6–7]. The coupling between the flow and thermal fields through the buoyancy force in a complex enclosure makes the mathematical model, which is used for study of the natural convection heat transfer, and is rather challenging. As a matter of fact, a group of reports concerned with the flow behavior and thermal performance in different kinds of enclosures have been presented. According to a literature survey, it is noted that the natural convective flows inside a rectangular enclosure were most extensively studied relatively. Numerous reports are available, for example, Ghia, et al. [8], presenting numerical solutions regarding natural convection in the rectangular enclosures. However, only a limited number of studies have been involved for the natural convection heat transfer in the complex enclosures. Among the limited studies, Lee et al. [9] performed numerical and experimental investigations of the effects of the fluid flow and heat transfer characteristics for natural convection inside a two dimensional rectangular enclosure with a semicircular or a triangular top wall. The results show that the average Nusselt of semicircular roof enclosure and triangular roof enclosure are greater than those of square enclosure by about 30% and 37%, respectively Cheng and Chao [10] predicted the buoyancy-driven flow in the annulus between two horizontal eccentric elliptical cylinders. Chang and Cheng [11] firstly dealt with a steady-state lid-driven forced convective flow in an arc-shaped cavity. It is found that in addition to the primary re-circulating vortex, a second vortex may be induced by the inertia or the buoyancy forces at a higher Reynolds number or Grashof number. Mahmud et al. [12] studied free convection in enclosures with vertical wavy walls. Recently, Chen and Cheng [13] studied steady natural convection in an inclined arc-shaped enclosure, both experimentally and numerically. Their reported results show that the features of the arc-shaped enclosure flows are rather different from those of the rectangular enclosure flows. However, in this study the attention is only focused on the heat transfer behavior of air.

On the other hand, the effects of inclination of the enclosure could be remarkable on the thermal performance. Ramos et al. [14] investigated numerically steady-state thermosyphon in a square enclosure, and authors found that the effects of the temperature differences and the inclination angle are significant on the flow pattern. Soong et al. [15] dealt with a numerical investigation of natural convection and the associated mode-transition and hysteresis phenomena in a two-dimensional differentially heated inclined enclosure. The present numerical study has demonstrated that the imperfection of the thermal boundary condition can be a trigger for the mode-transition.

☆ Communicated by Dr. W.J. Minkowycz.

* Corresponding author at: Department of Mechanical Engineering Lee Ming Institute of Technology, No.2-2, Lee-Juan Road, Tai-Shan, New Taipei City, 24305 Taiwan, ROC.

E-mail address: chencl@mail.lit.edu.tw (C.-L. Chen).

0735-1933/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2011.11.001

Furthermore, in addition to the three-dimensionality, the imperfection in thermal boundary condition, which is quite common in experiments, is one of the possible causes for the mismatch of the numerical prediction and the measured data. Adjlout et al. [16] have investigated numerical investigation of natural convection laminar flows in an inclined cavity with a wavy wall. The results obtained show that the hot the wall undulation affects the flow and the heat transfer rate in the cavity. Lately, natural convection heat transfer in the inclined cavities was also studied by Dalal et al. [17] and Bouali et al [18] for the boundary conditions having radiation or a spatially varying wall temperature. The obtained results all indicated the profound effects of inclination on the flow and thermal fields in the cavities.

Based on the existing information presented in these existing reports, it can be recognized that the inclination angle of a complex enclosure might have a profound influence on the natural convection heat transfer in the enclosures. However, information regarding the inclination effects on the heat convection in the complex enclosures is not sufficient. Moreover, to the authors' best knowledge, most of the existing reports are concerned with the gas fluids behavior, and the data about the inclination effects on the natural convection heat transfer with liquid flow in an arc-shaped enclosure are still not available. The related data, no matter theoretical or experimental, are essential to the cooling devices or lubrication units design since in these practical systems liquid fluids, such as water or lubrication oil, are commonly applied. Under these circumstances, the main objective of the present study is to examine the buoyancy-induced flow motion and natural convection heat transfer with liquid fluids contained in an inclined arc-shaped enclosure.

Schematic of the enclosure is shown in Fig. 1. The enclosure of width L is confined between an arc-shaped and a flat wall and is inclined at an angleθ. The profile of the arc-shaped wall is defined by the expression, $(x-p)^2+(y-q)^2=r^2$, which actually denotes a circle of radius r with the center located at point (p, q). In this analysis, the ratios p/r, q/r, and r/L are fixed at $1/2, 1/2\sqrt{3}$, and $1/\sqrt{3}$, respectively. The temperature of the arc-shaped wall is maintained at a higher level (T_H) and the temperature of the flat wall is maintained at a lower level (T_L). Two dimensionless parameters are varied in this study. The Grashof number (Gr) is varied in the range $1\times10^5 \leqq Gr \leqq 4\times10^6$, and the inclination angle(θ) is in the range $0° \leqq \theta \leqq 360°$. The Prandtl (Pr) number is assigned to be 4.0 for the liquid fluids.

2. Theoretical model

The flow and thermal fields are considered to be steady, laminar, incompressible, and two-dimensional. The fluid properties are assumed constant except for the variation of the density in the buoyancy terms of the momentum equations, which can be approximated by the Boussinesq assumption. The dimensionless stream function–vorticity formulation representing the laws of conservation in mass, momentum, and energy can be expressed in curvilinear coordinates as

$$\alpha\frac{\partial^2\Psi}{\partial\eta^2}-2\beta\frac{\partial^2\Psi}{\partial\xi\partial\eta}+\gamma\frac{\partial^2\Psi}{\partial\xi}=-J\Omega \tag{1}$$

$$J\left(\frac{\partial\Omega}{\partial\xi}\frac{\partial\Psi}{\partial\eta}-\frac{\partial\Omega}{\partial\eta}\frac{\partial\Psi}{\partial\xi}\right) = \left(\alpha\frac{\partial^2\Omega}{\partial\eta^2}-2\beta\frac{\partial^2\Omega}{\partial\xi\partial\eta}+\gamma\frac{\partial^2\Omega}{\partial\xi^2}\right)+JGr\left[\cos\theta\left(\frac{\partial Y}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial Y}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right)-\sin\theta\left(\frac{\partial X}{\partial\xi}\frac{\partial\theta}{\partial\eta}-\frac{\partial X}{\partial\eta}\frac{\partial\theta}{\partial\xi}\right)\right] \tag{2}$$

$$JPr\left[\left(\frac{\partial\Psi}{\partial\eta}\frac{\partial\theta}{\partial\xi}-\frac{\partial\Psi}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right)\right]=\left(\alpha\frac{\partial^2\theta}{\partial\eta^2}-2\beta\frac{\partial^2\theta}{\partial\xi\partial\eta}+\gamma\frac{\partial^2\theta}{\partial\xi^2}\right) \tag{3}$$

$$U=\frac{1}{J}\left(-\frac{\partial X}{\partial\eta}\frac{\partial\Psi}{\partial\xi}+\frac{\partial X}{\partial\xi}\frac{\partial\Psi}{\partial\eta}\right) \tag{4}$$

$$V=\frac{1}{J}\left(-\frac{\partial Y}{\partial\eta}\frac{\partial\Psi}{\partial\xi}+\frac{\partial Y}{\partial\xi}\frac{\partial\Psi}{\partial\eta}\right) \tag{5}$$

$$\Omega=\frac{1}{J}\left[\left(\frac{\partial Y}{\partial\eta}\frac{\partial V}{\partial\xi}-\frac{\partial Y}{\partial\xi}\frac{\partial V}{\partial\eta}\right)+\left(\frac{\partial X}{\partial\eta}\frac{\partial U}{\partial\xi}-\frac{\partial X}{\partial\xi}\frac{\partial U}{\partial\eta}\right)\right] \tag{6}$$

where

$$\alpha=\frac{\partial^2X}{\partial\xi^2}+\frac{\partial^2Y}{\partial\xi^2} \tag{7a}$$

$$\beta=\frac{\partial X}{\partial\xi}\frac{\partial X}{\partial\eta}+\frac{\partial Y}{\partial\eta}\frac{\partial Y}{\partial\xi} \tag{7b}$$

$$\gamma=\frac{\partial^2X}{\partial\eta^2}+\frac{\partial^2Y}{\partial\eta^2} \tag{7c}$$

$$J=\frac{\partial X}{\partial\xi}\frac{\partial Y}{\partial\eta}-\frac{\partial X}{\partial\eta}\frac{\partial Y}{\partial\xi} \tag{7d}$$

Note that J denotes the Jacobian of the coordinate mapping from the rectangular coordinates (X, Y) to the curvilinear coordinates (ξ, η). The dimensionless variables appearing in the above governing equations are defined by

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{u}{\nu/L},V=\frac{v}{\nu/L},\Psi=\frac{\varphi}{\nu},\Omega=\frac{\omega L^2}{\nu},\theta=\frac{T-T_L}{T_H-T_L}$$
$$Gr=\frac{g\bar{\beta}(T_H-T_L)L^3}{\nu^2},Pr=\frac{\nu}{\bar{\alpha}} \tag{8}$$

In addition, the boundary conditions associated with these equations are given as:

$$\text{On the flat wall}: U=0,V=0,\theta=0,\Psi=0,\text{and}\,\Omega=-\frac{1}{J}\left(\frac{\partial X}{\partial\xi}\frac{\partial U}{\partial\eta}\right) \tag{9a}$$

$$\text{On the arc-shape wall}: U=0,V=0,\theta=1,\Psi=0,\text{and}\,\Omega=-\frac{1}{J}\left(\frac{\partial Y}{\partial\xi}\frac{\partial V}{\partial\eta}+\frac{\partial X}{\partial\xi}\frac{\partial U}{\partial\eta}\right) \tag{9b}$$

The coordinate transformation functions $\xi=\xi(X,Y)$and$\eta=\eta(X,Y)$, denoting the curvilinear coordinate system, are obtained by using a numerical grid generation method similar to that described by Chen and Cheng [19]. A typical grid system of 101×101 grid points is adopted within the solution domain for computation. However, a careful check for the grid independence of the numerical solution has been made to ensure the accuracy and validity of the numerical

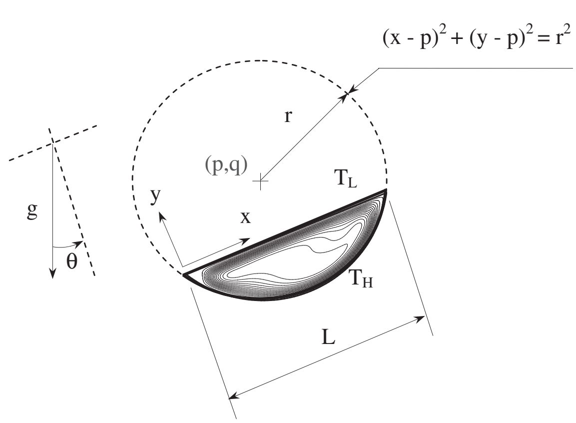

Fig. 1. Schematic diagram of an inclined arc-shaped enclosure.