International Communications in Heat and Mass Transfer 38 (2011) 1455–1462

Contents lists available at SciVerse ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Inverse estimation of boundary conditions on radiant enclosures by temperature measurement on a solid object ☆

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ARTICLE INFO

Available online 27 August 2011

Keywords:
Inverse heat transfer
Conduction heat transfer
Radiative heat transfer
Conjugate gradient method

ABSTRACT

In this paper, we present an inverse analysis to estimate the thermal boundary conditions over a two-dimensional radiant enclosure from the knowledge of the measured temperatures for some points on a solid object within the enclosure. The conduction heat transfer in the solid object and the radiative heat transfer between the surface elements of the enclosure are formulated by the finite volume method and the net radiative method, respectively. The resultant set of nonlinear equations is solved by the Newton's method. The inverse problem for estimation of boundary conditions over the radiant enclosure is solved by the conjugate gradient method.

1. Introduction

Inverse heat transfer problems are concerned with the determination of the thermal properties, the initial condition, the boundary conditions, and the strength of heat source from the knowledge of the temperature or heat flux measurements taken at the interior or the boundary points of the domain. They have been widely used in many design and manufacturing applications, especially when direct measurements of the surface condition are not possible. Many studies of the inverse problems with conduction, convection and radiation have been reported [1–16]. Inverse problems have also received much attention in recent years for the cases with multimode heat transfer [17–22]. A comprehensive study of inverse heat transfer problems has been reported in [23].

In the present work, we deal with the inverse problem of estimating the boundary conditions over the boundary surface of a radiant enclosure by measuring the temperatures of some points on a solid object within the enclosure. The applications may be seen in manufacturing, thermal treatment and food industries where we are interested to know the strength of radiant heaters located on the wall surface of the radiant oven by the measurement of temperatures over some points of product surface. Heat is transferred in the solid object by conduction, and the dominant mode of heat transfer in the enclosure is radiation. The solid object is subdivided into control volumes and the boundary surface of the solid object and the radiant enclosure are subdivided into surface elements. For the direct problem, the conduction heat transfer in the solid object is formulated by the finite volume method, and the radiation heat transfer between surface elements are formulated by the net radiation method. The complete set of nonlinear equations is then solved by Newton's method. For the inverse problem, the temperature distribution over some parts of boundary surface of the radiant enclosure is regarded as unknown, and the temperatures for some sampling points over the solid object are considered to be available by the measurement. The conjugate gradient method is used for minimization of the objective function which is defined as the sum of square deviations between the measured and estimated values of temperatures on the solid object. Finally, the performance and the accuracy of the present method for recovering the boundary temperature distribution over the radiant enclosure from the knowledge of measured temperatures of solid object is examined by considering some examples with different temperature distributions over the radiant enclosure. The effects of the location of solid object within the enclosure, and noisy input data on the accuracy of the inverse solution are investigated by several numerical experiments.

2. Description of problem

Consider a two-dimensional square enclosure A, and the square solid object B within it, as depicted in Fig. 1. All the internal walls of the enclosure A, and the boundary surfaces of the solid object B are diffuse-gray. The enclosure filled with a transparent medium. All the thermal properties are assumed to be constant. Heat is transferred by radiation throughout the enclosure A, and is transferred in the solid object B with conduction. The side walls of the enclosure are at constant temperature of $T_s = 300\,K$ and the top wall of the enclosure is kept insulated. No boundary condition is specified over the bottom wall of the enclosure. The aim of the inverse problem is to find the boundary conditions over the bottom wall of the enclosure

☆ Communicated by W.J. Minkowycz.

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doi:10.1016/j.icheatmasstransfer.2011.08.015

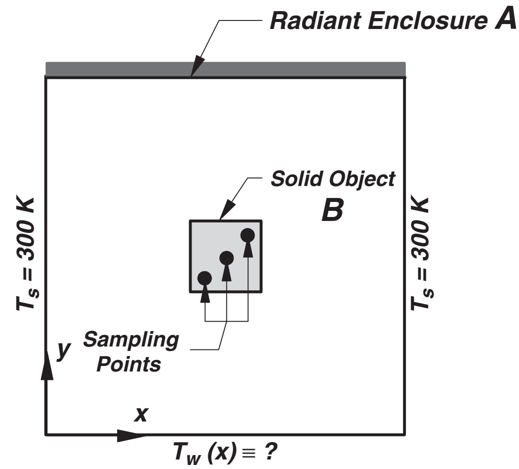

Fig. 1. Two-dimensional square enclosure A, and the square solid object B within it.

from the knowledge of measured temperatures for some sampling points on the solid object B.

3. Direct problem

The steady state conduction heat transfer in the square solid object B is governed by the Fourier's law of conduction as follows:

$$\nabla\cdot(\kappa\nabla T) = 0 \tag{1}$$

where κ and T are the conductivity and temperature, respectively. Eq. (1) is solved by the finite volume method. In this method, the domain of interest is subdivided into V finite volumes. Writing energy balance for all finite volumes leads to a set of V linear algebraic equations, which can be solved by conventional solvers such as LU decomposition approach, provided that the boundary conditions over all boundary surfaces are known. However, here no boundary conditions are known a priori. Hence, the number of unknowns is $V+Z$, where Z is the number of surface elements over the boundary surface of the solid object B.

The radiative exchange for surface elements can be described by the following equation [24]

$$\sum_{j=1}^{R}\left(\frac{\delta_{kj}}{\varepsilon_j}-\frac{1-\varepsilon_j}{\varepsilon_j}F_{k-j}\right)q_j = \sum_{j=1}^{R}\left(\delta_{kj}-F_{k-j}\right)\sigma T_j^4 \ , \ 1\le k\le R \tag{2}$$

where T_j, q_j, and ε_j are temperature, heat flux and emissivity of the element surface j. Here R is the total number of radiative surface elements in the enclosure, F_{k-j} is the geometric configuration factor which can be calculated by the Hottel's crossed-string method [25], and δ_{kj} is the Kronicker delta defined by

$$\delta_{kj} = \begin{cases} 1 & k=j \\ 0 & k\neq j \end{cases} \tag{3}$$

The boundary conditions over the boundary surfaces of the enclosure may be written as follows:

$$q(x,1) = 0.0 \tag{4a}$$

$$T(0,y) = T(1,y) = T_s \tag{4b}$$

$$T(x,0) = \sum_{m=1}^{M}\varphi_m u(x-x_m)u(x_{m+1}-x) \tag{4c}$$

Where M is the number of surface elements over the bottom wall, φ_ms are the known coefficients, and $u(x-x_m)$ is the well known unit step function defined as

$$u(x-x_m) = \begin{cases} 1 & x\ge x_m \\ 0 & x<x_m \end{cases} \tag{5}$$

If the elemental temperatures over the boundary surfaces of the solid object B are specified, then the set of R Eq. (2) can be solved to calculate the unknown elemental temperatures or heat fluxes. However, since no boundary condition is known over the boundary surfaces of the solid object B, the number of unknowns is increased to $R+Z$.

As discussed above, we conclude that the conduction heat transfer equation through the solid object medium and the radiative transfer equation in the enclosure cannot be solved independently. Considering the interface condition over all the boundary surface elements of the solid object made by the following equation

$$\kappa\frac{\partial T_z}{\partial n_z} + q_z = 0 \ , \ z = 1,\ldots,Z \tag{6}$$

where n_z is the normal direction to the boundary surface element z, we now have a set of total $V+R+Z$ nonlinear equations with the same number of unknowns. Because of nonlinearity due to the fourth power of temperatures in the radiative transfer equation, the conventional solvers of linear set of algebraic equations cannot be used for solving the set of equations. Hence, the set of equations must be solved through an iterative approach, such as Newton's method. The Newton's method for solving the set of nonlinear equations is described in detail in Ref. [26], and will not be repeated.

4. Inverse problem

For the inverse problem, the temperature distribution over the bottom wall of the radiant enclosure is regarded as unknown, and the measured values of temperatures for some sampling points are available for the analysis. The objective function is expressed by the sum of square residuals between the estimated and measured values of temperatures for sampling points over the solid object as follows:

$$f(\mathbf{\Phi}) = \left[\mathbf{Y}-\mathbf{T}(\mathbf{\Phi})\right]^{\mathrm{T}}\left[\mathbf{Y}-\mathbf{T}(\mathbf{\Phi})\right] \tag{7}$$

where $\mathbf{\Phi}=\{\varphi_1,\ldots,\varphi_M\}$ is the vector of unknown parameters. $\mathbf{Y}=\{Y_1,\ldots,Y_N\}$ and $\mathbf{T}(\mathbf{\Phi})=\{\mathbf{T}_1(\mathbf{\Phi}),\ldots,\mathbf{T}_N(\mathbf{\Phi})\}$ are the vectors of measured and estimated temperatures of sampling points on the solid object, respectively. M and N are the number of elements over the bottom wall of the enclosure and the number of sampling points on the solid object, respectively.

The CGM is an iterative procedure in which at each iteration a suitable step size, β, is taken along a direction of descent, $\mathbf{d}$, in order to minimize the objective function, so that

$$\mathbf{\Phi}^{\nu+1} = \mathbf{\Phi}^{\nu}-\beta^{\nu}\mathbf{d}^{\nu} \tag{8}$$

where the superscript ν is the iteration number. The direction of descent can be determined as a conjugation of the gradient direction, ∇f, and the direction of descent from the previous iteration as follows:

$$\mathbf{d}^{\nu} = \nabla f(\mathbf{\Phi}^{\nu}) + \gamma^{\nu}\mathbf{d}^{\nu-1} \tag{9}$$