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Numerical simulation of natural convection in an open-ended square cavity filled with porous medium by lattice Boltzmann method ☆

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ABSTRACT

In the present work, natural convection in an open-ended square cavity packed with porous medium is simulated. The double-population approach is used to simulate hydrodynamic and thermal fields, and the Taylor series expansion and the least-squares-based lattice Boltzmann method has been implemented to extend the thermal model. The effect of a porous medium is taken into account by introducing the porosity into the equilibrium distribution function and adding a force term to the evolution equation. The Brinkman–Forchheimer equation, which includes the viscous and inertial terms, is applied to predict the heat transfer and fluid dynamics in the non-Darcy regime. The present model is validated with the previous literature. A comprehensive parametric study of natural convective flows is performed for various values of Rayleigh number and porosity. It is found that these two parameters have considerable influence on heat transfer.

1. Introduction

Recently, many researchers have made endeavors to enhance the ability of the lattice Boltzmann method (LBM) to simulate thermo-fluidics. Three methods for solving thermal LBM exist: the multispeed, the passive scalar and the double-population approaches. Since the LBM suffered from the instability, He et al. [1] developed the double-population approach to overcome this drawback. Also, they proposed Taylor series expansion and the least squares approach to employ the LBM more efficiently for flows with arbitrary geometry.

Natural convection has lots of applications in both nature and engineering, such as the cooling of electronic devices and heat transfer improvement in heat exchanger apparatuses and petroleum reservoirs. Because cavities and slots are benchmark test cases, many numerical works on open cavities have been performed [2,3]. Similarly, several experimental studies have been done on open cavities with aspect ratio of unity [4,5].

Research on natural convection in enclosure packed with a porous medium is motivated by its wide applications in engineering, such as drying processes, chemical catalytic reactors and solar power collectors. Several models have been introduced for natural convection heat transfer in porous media. An excellent and comprehensive review has been given by Nield and Bejan [6]. The buoyancy-driven convection in an open-ended cavity with an obstructing medium such as a porous material is analyzed by Ettefagh J. and Vafai K. [7]. Saeid N.H. and Pop I. [8] studied numerically the steady natural convection in a square cavity filled with a porous medium with the Darcy–Forchheimer model, and Nithiarasu et al. [9] also solved this problem with the Brinkman–Forchheimer equation using conventional numerical methods and demonstrated that the equation can appropriately predict the heat transfer and fluid dynamics in the non-Darcy regime.

Natural convection in an open-ended cavity using LBM was modeled by A.A Mohamad and his colleagues [10]. In their work, D2Q9 for flow and D2Q4 for temperature were utilized, and the influences of Rayleigh number and aspect ratio of the cavity have been investigated in the range of 10^4–10^6 and of 0.5–10, respectively. They concluded that heat transfer increases with increasing Ra and decreases asymptotically with increasing the aspect ratio.

LBM has been applied successfully to simulate fluid flow in porous media. Takeshi Seta et al. [11] analyzed the thermal performance of natural convection in the square cavity with the presence of a porous medium for different values of the Rayleigh number, the Darcy number and porosity. They investigated the capability of LBM to solve this problem and observed a good agreement between their model and earlier studies. Wei-Wei Yan et al. [12] implemented LBM in a natural convection problem in a square cavity filled with a heterogeneously porous medium. It was reported that the porosity near the walls has a significant influence on heat transfer while the porosity in the middle of the cavity has little effect on the natural convection.

To the best knowledge of the authors of this work, the problem of natural convection in an open-ended cavity filled with a porous medium using the thermal lattice Boltzmann method (TLBM) has never been studied. The aim of the present study is to evaluate heat transfer parameters and flow characteristics by altering the porosity and Rayleigh number. The numerical results in the present work

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Nomenclature

c	lattice streaming speed
c_s	speed of sound
c_0	parameter defined in Eq. (21)
c_1	parameter defined in Eq. (22)
Da	Darcy number, $=K/L^2$
e_α	particle velocity vector
F	total body force vector
F_α	discrete body force in LBM
f_α	density distribution function
f_α^{eq}	equilibrium density distribution function
G	buoyancy force term per unit mass
g_α	internal energy distribution function
g_α^{eq}	equilibrium internal energy distribution function
g_0	acceleration due to gravity
j	unit vector in the *y*-direction
K	permeability of porous media
L	height and width of the cavity
$\overline{Nu}$	average Nusselt number
P	pressure
Pr	Prandtl number
R	gas constant
Ra	Rayleigh number
T	temperature
T_m	average temperature
T_w	left-wall temperature
T_a	ambient temperature
u	fluid velocity vector
v	temporal velocity

Greek symbols

α	thermal conductivity
β	thermal coefficient expansion
δt	time step in the lattice
δx	space step in the lattice
ϵ	porosity of the medium
ξ	microscopic velocity
ρ	fluid density
ρ_m	average fluid density
τ_υ	relaxation time for the density distribution function
τ_c	relaxation time for the internal energy distribution function
υ	kinetic viscosity
υ_e	effective viscosity
χ	thermal diffusivity
ω_α	weighting coefficient

Subscripts

α	discrete speed directions ($\alpha=0,\ldots,8$)
w	left wall
a	ambient

Superscripts

eq	equilibrium

demonstrate great change in the flow and temperature fields, with variations of the parameters mentioned above. Moreover, as these parameters are varied in a wide range, some new phenomena are observed.

First, a concise description of the governing equations and the numerical strategy are represented. Next, the velocity and temperature boundary conditions applied for open and solid boundaries are introduced. Then, results and discussion are reported, and finally, some conclusions are drawn.

2. Numerical method

The continuity, the Brinkman–Forchheimer, and the energy equations are respectively written as

$$\nabla.\mathbf{u}=0 \tag{1}$$

$$\partial_t\mathbf{u}+(\mathbf{u}.\nabla)\left(\frac{\mathbf{u}}{\varepsilon}\right)=-\frac{1}{\rho}\nabla(\varepsilon p)+\upsilon_e\nabla^2\mathbf{u}+\mathbf{F} \tag{2}$$

$$\partial_t(\rho e)+\nabla.(\rho\mathbf{u}e)=\nabla\chi^2(\rho e) \tag{3}$$

where **u** is the fluid velocity vector, ε is the porosity of the medium, υ_e is the effective viscosity, ρ is the density, p is the pressure, and χ is the thermal diffusivity. With the widely used Ergun's relation [13], **F** is the body force that denotes the viscous diffusion, the inertia due to the presence of a porous medium and an external force, i.e.,

$$\mathbf{F}=-\frac{\varepsilon\upsilon}{K}\mathbf{u}-\frac{1.75}{\sqrt{150\,\varepsilon K}}|\mathbf{u}|\mathbf{u}+\varepsilon\mathbf{G}, \tag{4}$$

where υ is the kinematic viscosity, **G** is the buoyancy force vector, and K is the permeability of porous media. With the Boussinesq approximation, all the fluid properties are considered constant, except in the buoyancy force term, where the fluid density is given by $\rho=\rho_m\,[1-\beta(T-T_m)]$. ρ_m is the average fluid density, T_m is the average fluid temperature and β is the thermal coefficient expansion of the fluid. In this case, the buoyancy force acting per unit mass is $\rho G=\rho g_0\beta\,(T-T_m)\,\mathbf{j}$, where **j** is in a direction opposite to gravity and g_0 is the acceleration due to gravity.

The lattice Boltzmann method provides an alternative way to solve the partial differential equations by evolving variables on a set of lattices. The mathematical demonstration of the TLBM can be found in He et al. [1].

The main hypotheses of this model are:

- The Bhatnagar, Gross and Krook approximation (BGK) results that the collision operator is expressed as a single relaxation time to the local equilibrium.
- The Knudsen number is assumed to be a small parameter.
- The flow is incompressible.
- The viscous heat dissipation and compression work done by pressure are neglected.

The evolution of the density distribution function $\tilde{f}$ for a single fluid particle is then given by

$$\frac{D\tilde{f}}{Dt}=\partial_t\tilde{f}+(\xi.\nabla)\tilde{f}=-\frac{\tilde{f}-\tilde{f}^{eq}}{\tau_v}+\mathbf{F} \tag{5}$$

where ξ is the microscopic velocity, τ_υ is the relaxation time for the density distribution function ,$\tilde{f}^{eq}$ is the Maxwell–Boltzmann equilibrium distribution function and **F** refers to the body forces. Similarly, the internal energy distribution function $\tilde{g}$ is given by the following evolution equation:

$$\frac{D\tilde{g}}{Dt}=\partial_t\tilde{g}+\left(\vec{\xi}.\nabla\right)\tilde{g}=-\frac{\tilde{g}-\tilde{g}^{eq}}{\tau_c} \tag{6}$$