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Laminar heat transfer in a porous channel simulated with a two-energy equation model ☆

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ABSTRACT

Laminar heat transfer in a porous channel is numerically simulated with a two-energy equation model for conduction and convection. Macroscopic equations for continuity, momentum and energy transport for the fluid and solid phases are presented. The numerical methodology employed is based on the control volume approach with a boundary-fitted non-orthogonal coordinate system. Fully developed forced convection in a porous channel bounded by parallel plates is considered. Solutions for Nusselt numbers along the channel are presented for laminar flows. Results simulate the effects Reynolds number *Re*, porosity, particle size and solid-to-fluid thermal conductivity ratio on Nusselt sumber, *Nu*, which is defined for both the solid and fluid phases. High *Re*, low porosities, low particle diameters and low thermal conductivity ratios promote thermal equilibrium between phases leading to higher values of *Nu*.

1. Introduction

The assumption of local thermal equilibrium when analyzing heat transport in porous media requires several constraints which have been investigated in the literature [1–5]. For example, this condition is no longer valid when the particles or pores are not small enough, when the thermal properties differ widely, or when convective transport is not important. Furthermore, when there is a significant heat generation in any of the phases, the system will depart rapidly from the local thermal equilibrium state [6]. For such extreme conditions, the one-energy equation or one-temperature model is inadequate to correctly describe both the transients associated with the quench front penetrating the hot dry porous layer and regions where dry out occurs. When the assumption of local thermal equilibrium fails to be valid, one possible solution is to develop separate transport equations for each phase [7–9]. This leads to mathematical models that are referred to as thermal non-equilibrium models, which consider distinct energy equations for each phase. However, analyses of heat transfer in porous media based on two-equation models are more complex because they require information on interstitial heat transfer between phases as well as the interfacial surface area. Due to such requirement, investigators have worked on how to obtain the interfacial heat transfer coefficient. Examples of such efforts are found in the work of Wakao et al. [10], who obtained a heuristic correlation for closely packed bed and compared their results with experimental data. Also found in the literature is a numerical correlation for the interfacial convective heat transfer coefficient, which was proposed by Kuwahara et al. [11] for laminar flow and was based on porosity dependency.

In previously published articles, a mathematical model for predicting turbulent flow in porous media has been presented [12], including buoyant flows [13,14] as well as channel flows through porous inserts [15], perforated baffles [16] and across macroscopic interfaces [17]. In all of the above, the so-called one-energy equation model was used, which invoked the local thermal equilibrium between the working fluid and solid matrix. Later, Saito and de Lemos [18] presented simulations for laminar flows thorough the void space of rods, which were arranged in arrays and simulated a repetitive unit cell in a model of a porous medium. In a following article [19], a proposition of a correlation for the interfacial heat transfer coefficient for turbulent flow in a packed bed was presented. Results in [18,19] contributed to the development of a macroscopic model for non-equilibrium heat transfer in porous media, but no results for macroscopic flow were presented.

The purpose of this contribution is to combine the flow [12] and thermal non-equilibrium [19] models for porous media and predict macroscopic forced convection in a porous channel bounded by parallel plates.

2. Macroscopic transport

2.1. Flow equations

Macroscopic equations obtained after volume integration over a Representative Elementary Volume (REV) are given as [20,21],

$$\text{Continuity}: \nabla \cdot \mathbf{u}_D = 0. \tag{1}$$

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Nomenclature

Latin characters

Ai	interface area between fluid and solid phases
c_F	Forchheimer coefficient
c_p	fluid specific heat
D	particle diameter
Da	Darcy number, $Da = K/H^2$
h_i	interfacial heat transfer coefficient
$\mathbf{I}$	unit tensor
K	permeability
k_f	fluid thermal conductivity
k_s	solid thermal conductivity
$\mathbf{K}_{disp}$	dispersion tensor
$\mathbf{K}_{f,s}$	thermal conductivity tensor for fluid phase.
$\mathbf{K}_{s,f}$	thermal conductivity tensor for solid phase.
p	pressure
Pr	$Pr = \nu/\alpha$, Prandtl number
Re_D	Reynolds number based on D and superficial velocity $\mathbf{u}_D$
T	temperature
T_{ms}	average temperature of solid phase
T_{mf}	average temperature of fluid phase
u_B	bulk velocity
$\mathbf{u}$	local velocity
$\mathbf{u}_D$	Darcy or superficial velocity (volume average of $\mathbf{u}$)
x, y	Cartesian coordinates, m
X, Y	non-dimensional coordinates, x/H and y/H

Greek characters

α	fluid thermal diffusivity
ΔV	representative elementary volume
ΔV_f	fluid volume inside ΔV
μ	fluid dynamic viscosity
ν	fluid kinematic viscosity
ρ	fluid density
ϕ	$\phi = \Delta V_f/\Delta V$, porosity

Subscripts

w	wall
s	solid phase
f	fluid phase

$$\text{Momentum}: \rho\left[\frac{\partial \mathbf{u}_D}{\partial t} + \nabla\cdot\left(\frac{\mathbf{u}_D\mathbf{u}_D}{\phi}\right)\right] = -\nabla\left(\phi\langle\overline{p}\rangle^i\right) + \mu\nabla^2\mathbf{u}_D - \left[\frac{\mu\phi}{K}\mathbf{u}_D + \frac{c_F\phi\rho|\mathbf{u}_D|\mathbf{u}_D}{\sqrt{K}}\right], \tag{2}$$

where the last two terms in Eq. (2) represent the Darcy and Forchheimer contributions. The symbol K is the porous medium permeability, c_F is the form drag or Forchheimer coefficient, $\langle\overline{p}\rangle^i$ is the intrinsic average pressure of the fluid and ϕ is the porosity of the porous medium. In this work, the permeability is taken as a function of the particle diameter D as [22],

$$K = \frac{\phi^3 D^2}{144(1-\phi)^2}. \tag{3}$$

2.2. Energy equations

A two-energy equation model for convection and conduction in porous media, considering a heat transfer coefficient between the fluid and the solid phases, is given by the following equation set:

$$\left(\rho c_p\right)_f\left[\frac{\partial\phi\langle T_f\rangle^i}{\partial t} + \nabla\cdot\left\{\phi\left(\langle\mathbf{u}\rangle^i\langle T_f\rangle^i + \langle^i\mathbf{u}^i T_f\rangle^i\right)\right\}\right] = \nabla\cdot\left[k_f\nabla\left(\phi\langle T_f\rangle^i\right)\right] + \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_f\nabla T_f dA \tag{4}$$

$$\left(\rho c_p\right)_s\left\{\frac{\partial(1-\phi)\langle T_s\rangle^i}{\partial t}\right\} = \nabla\cdot\left\{k_s\nabla\left[(1-\phi)\langle T_s\rangle^i\right]\right\} - \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_s\nabla T_s dA \tag{5}$$

where, $\langle T_s\rangle^i$ and $\langle T_f\rangle^i$ denote the intrinsic average temperature of solid and fluid phases, respectively, A_i is the interfacial area within the REV. The convective transport is described in the second term on the right hand side of Eq. (4) (see Rocamora and de Lemos [23] for details).

2.2.1. Interfacial heat transfer

In Eqs. (4) and (5) the heat transferred between the two phases can be modeled by means of a film coefficient h_i such that,

$$h_i a_i\left(\langle T_s\rangle^i - \langle T_f\rangle^i\right) = \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_f\nabla T_f dA = \frac{1}{\Delta V}\int_{A_i}\mathbf{n}_i\cdot k_s\nabla T_s dA \tag{6}$$

where, $a_i = A_i/\Delta V$ is the interfacial area per unit volume. In porous media, the high values of a_i make them attractive for transferring thermal energy via conduction through the solid followed by convection to a fluid stream.

As mentioned earlier, Wakao et al. [10] obtained a correlation for closely packed bed of particle diameter D and compared their results with experimental data. This correlation for the interfacial heat transfer coefficient is given by,

$$\frac{h_i D}{k_f} = 2 + 1.1 Re_D^{0.6} Pr^{1/3}. \tag{7}$$

Further, a numerical correlation for the interfacial convective heat transfer coefficient was proposed by Kuwahara et al. [11] for laminar flow as,

$$\frac{h_i D}{k_f} = \left(1 + \frac{4(1-\phi)}{\phi}\right) + \frac{1}{2}(1-\phi)^{1/2} Re_D Pr^{1/3}, \quad \text{valid for } 0.2 < \phi < 0.9. \tag{8}$$

Eq. (8) is based on porosity dependency and is valid for packed beds of particle diameter D. In addition, Saito and de Lemos [18] numerically calculated the interfacial heat transfer coefficient h_i for laminar flow through an infinite rod array. In their physical model, the porous medium was considered to be formed by a large number of regularly arranged solid square rods. This same methodology was applied by Saito and de Lemos [19], who proposed a correlation for h_i for turbulent flow as,

$$\frac{h_i D}{k_f} = 0.08\left(\frac{Re_D}{\phi}\right)^{0.8} Pr^{1/3}; \quad \text{for} \quad 1.0\times10^4 < \frac{Re_D}{\phi} < 2.0\times10^7, \quad \text{valid for } 0.2 < \phi < 0.9. \tag{9}$$

2.2.2. Thermal dispersion

In order to apply Eq. (4) to obtain the fluid temperature field in porous media, the thermal dispersion term, 3rd term on the r.h.s of