International Communications in Heat and Mass Transfer 36 (2009) 1025–1029

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Natural convection boundary layer on a horizontal elliptical cylinder with constant heat flux and internal heat generation ☆

Ching-Yang Cheng

Department of Mechanical Engineering, Southern Taiwan University, Yungkang 71005, Taiwan

ARTICLE INFO

Available online 12 August 2009

Keywords:
Heat transfer
Elliptical cylinder
Natural convection
Heat generation
Constant heat flux

ABSTRACT

In this paper the natural convection boundary layer on a horizontal elliptical cylinder with constant heat flux and temperature dependent internal heat generation is investigated. The mathematical problem is reduced to a pair of coupled partial differential equations for the temperature and the stream function, and the resulting nonlinear equations are solved numerically by cubic spline collocation method. Results for the local Nusselt number and the local skin-friction coefficient are presented as functions of eccentric angle for various values of heat generation parameters, Prandtl numbers and aspect ratios. An increase in the aspect ratio of the elliptical cylinder decreases the average surface temperature of the elliptical cylinder with blunt orientation, while it increases the average surface temperature of the elliptical cylinder with slender orientation. Moreover, an increase in the heat generation parameter for natural convection flow over a horizontal elliptic cylinder with constant heat flux leads to an increase in the average surface temperature of the elliptical cylinder.

1. Introduction

Natural convection heat transfer and flow around heated, horizontal cylinders for many different kinds of fluids are of great importance because of their extensive industrial applications. These applications have motivated extensive research on the hear transfer and flow characteristics driven by natural convection from horizontal cylinders. Saville and Churchill [1] studied the laminar natural convection boundary-layer flow near horizontal cylinders and vertical axisymmetric bodies. Merkin [2] examined the natural convection boundary layer flow over a cylinder of elliptic cross-section. Lien et al. [3] studied the free convection heat transfer of micropolar fluid near a horizontal permeable cylinder at a non-uniform thermal condition. Bhattacharyya and Pop [4] studied the free convection heat transfer from an elliptical cylinder in micropolar fluids. Hossain et al. [5] studied the effect of thermal radiation on natural convection over cylinders of elliptic cross section. Mansour et al. [6] examined the coupled heat and mass transfer in magnetohydrodynamic flow of micropolar fluid on circular cylinders with uniform heat and mass flux. Cheng [7] studied the natural convection heat and mass transfer from a horizontal cylinder of elliptic cross section in micropolar fluid. Moreover, Cheng [8] studied the effect of temperature-dependent viscosity on the natural convection heat transfer from a horizontal isothermal cylinder of elliptic cross section.

Possible heat generation effects may alter the temperature distribution. This may occur in such applications related to nuclear reactor cores, fire and combustion modelling, electronic chips and semiconductor wafers. It is necessary to take into account the effect of heat generation to obtain a better estimation of the flow and heat transfer behavior. Vajravelu and Hadjinicolaou [9] examined the heat transfer characteristics in a laminar boundary layer flow of a viscous fluid over a linearly stretching continuous surface with viscous dissipation/frictional heating and internal heat generation. Chamkha and Issa [10] studied the effect of the heat generation or absorption and thermophoresis on a hydromagnetic flow with heat and mass transfer over a flat plate. Mendez and Trevino [11] examined the effects of the conjugate conduction-natural convection heat transfer along a thin vertical plate with non-uniform heat generation. Molla et al. [12] investigated the natural convection flow with heat generation/absorption along a uniform heated vertical wavy surface. Molla et al. [13] studied the natural convection flow from an isothermal horizontal circular cylinder in the presence of heat generation. Mamun et al. [14] studied the MHD-conjugate heat transfer for a vertical flat plate in the presence of viscous dissipation and heat generation. Moreover, Molla et al. [15] examined the natural convection flow from a horizontal cylinder with uniform heat flux in presence of heat generation.

Motivated by the works of Molla et al. [13,15], this work uses a suitable coordinate transformation and the cubic spline collocation method to study the natural convection heat transfer from a horizontal cylinder of elliptic cross section in a Newtonian fluid with constant heat flux and temperature dependent internal heat generation. The results obtained herein are compared with the solutions obtained by Merkin [2] for natural convection heat transfer from an elliptical cylinder in Newtonian fluids with constant heat flux to assess the accuracy of the solutions. This work studies the influence of the heat

☆ Communicated by W.J. Minkowycz.

E-mail address: cycheng@mail.stut.edu.tw.

0735-1933/$ – see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.icheatmasstransfer.2009.07.007

Nomenclature

A	Angle between outward normal and downward vertical
a	Length of semi-major axis of ellipse
b	Length of semi-minor axis of ellipse
C_f	Local skin-friction coefficient
e	Eccentricity
f	Reduced stream function
g	Gravitational acceleration
G	Heat generation parameter
Gr	Grashof number
h	Heat transfer coefficient
k	Thermal conductivity
Nu	Nusselt number
Pr	Prandtl number
q_w	Surface heat flux
Q_0	Heat generation constant
T	Temperature
U	Characteristic velocity
u, v	Velocity components
x, y	Cartesian coordiates

Greek symbols

α	Thermal diffusivity
β	Coefficient of thermal expansion
ξ, η	Dimensionless coordinates
θ	Dimensionless temperature
$\bar{\theta}_w$	Dimensionless average surface temperature
μ	Viscosity
ρ	Density
ψ	Dimensionless stream function
$\bar{\psi}$	Stream function
Ω	Eccentric angle
τ_w	Wall shear stress

Subscripts

w	Condition at wall
∞	Condition at infinity

generation parameter, the aspect ratio, the Prandtl number and the orientation on the heat transfer and flow characteristics near a horizontal cylinder of elliptic cross section in Newtonian fluids with constant heat flux and temperature dependent internal heat generation. The solutions given in this work include the results in both cases when the major axis is horizontal (blunt orientation) and vertical (slender orientation).

2. Analysis

Consider the steady-state two-dimensional, laminar natural convection boundary layer flow near a horizontal elliptical cylinder in a Newtonian fluid with temperature dependent internal heat generation, as shown in Fig. 1, where a is the length of semi-major axis, b is the length of semi-minor axis for the elliptical cylinder. In Fig. 1, A denotes the angle made by the outward normal from the cylinder with the downward vertical and Ω is the eccentric angle. Note that this work investigates horizontal elliptical cylinders of two different orientations. The orientation is blunt when the major axis is horizontal, as shown in Fig. 1, and the orientation is slender when the major axis is vertical. The surface of the elliptical cylinder is held at a constant heat

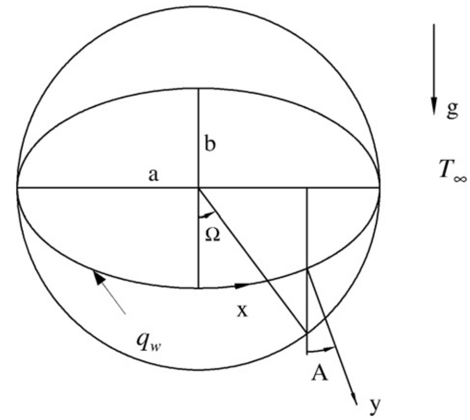

Fig. 1. Physical model and coordinate system for an elliptical cylinder of blunt orientation.

flux q_w while the ambient fluid temperature is T_∞. The fluid properties are assumed to be constant except for density variations in the buoyancy force term.

Based on the boundary-layer and Boussinesq approximations, the governing for laminar boundary-layer flow by natural convection of a Newtonian fluid with temperature dependent internal heat generation near a horizontal cylinder of elliptical cross section can be written in two-dimensional Cartesian coordinates (x, y) as [2]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \rho g\beta(T - T_\infty)\sin A \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho C_P}(T - T_\infty) \tag{3}$$

The appropriate boundary conditions for the problem are:

$$u = v = 0, \ -k\frac{\partial T}{\partial y} = q_w \quad \text{on} \quad y = 0 \tag{4}$$

$$u = 0, T = T_\infty \quad \text{as} \quad y \to \infty \tag{5}$$

Here u and v are the velocity components in the x and y directions, respectively. T is the fluid temperature Furthermore, α and β are the thermal diffusivity and the coefficient for thermal expansion, respectively. g is the gravitational acceleration. Property μ is the dynamic fluid viscosity and ρ is the fluid density. C_P is the specific heat at constant pressure and k is the thermal conductivity. Moreover, Q_0 is a heat generation constant which may be either positive or negative. This source term represents the heat generation when $Q_0>0$ and the heat absorption when $Q_0<0$.

After introducing the stream function $\bar{\psi}$ to satisfy the relations: $u=\partial\bar{\psi}/\partial y$ and $v=-\partial\bar{\psi}/\partial x$, and introducing the Grashof number $Gr=\rho^2 g\beta q_w a^4/(k\mu^2)$, we then define the nondimensional variables:

$$\xi = x/a, \ \eta = (y/a)Gr^{1/5}, \ \psi = (\bar{\psi}\rho/\mu)Gr^{-1/5}, \tag{6}$$

$$\theta = (T - T_\infty)Gr^{1/5}/(aq_w/k)$$

Substituting the variables defined in Eq. (6) into Eqs. (1)–(3) leads to the following equations:

$$\frac{\partial\psi}{\partial\eta}\frac{\partial^2\psi}{\partial\eta\partial\xi} - \frac{\partial\psi}{\partial\xi}\frac{\partial^2\psi}{\partial\eta^2} = \frac{\partial^3\psi}{\partial\eta^3} + \theta\sin A \tag{7}$$