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# Stagnation point flow and heat transfer over a stretching/shrinking sheet in a porous medium ☆

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ABSTRACT

The steady stagnation point flow and heat transfer over a shrinking sheet in a porous medium is studied. A similarity transformation is used to reduce the governing system of partial differential equations to a set of nonlinear ordinary differential equations which are then solved numerically using the Keller-box method. The behavior of the flow and heat transfer characteristics for different values of the governing parameters are analyzed and discussed. Results for the skin friction coefficient, local Nusselt number, velocity profiles as well as temperature profiles are presented for different values of the governing parameters. The results indicate that dual solutions exist for the shrinking case.



## 1. Introduction

Porous materials such as sand and crushed rock underground are saturated with water which, under the influence of local pressure gradients, migrate and transport the liquid through the material. The transport properties of fluid-saturated porous materials are very important in the petroleum and geothermal industries. Further examples of convection through porous media may be found in manmade systems such as fiber and granular insulations, winding structures for high-power density electric machines, and the cores of nuclear reactors (Bejan [1]), food processing and storage, thermal insulation of buildings, geophysical systems, electro-chemistry, metallurgy, the design of pebble bed nuclear reactors, underground disposal of nuclear or non-nuclear waste, cooling system of electronic devices, etc. Excellent reviews of the topic can be found in the books by Nield and Bejan [2], Pop and Ingham [3], Bejan et al. [4], Ingham and Pop [5], Vafai [6], Vadasz [7] and Vafai [8]. Vafai and Tien [9] analyzed the effects of a solid boundary and the inertial forces on flow and heat transfer through a porous medium and reported that the inertia effects increase with the higher permeability and the lower fluid viscosity. The steady stagnation point flow through a porous medium bounded by a vertical surface was investigated by Ishak et al. [10] and it was found that dual solutions exist for both assisting and opposing flows.

Viscous fluid motion toward a stagnation point on a solid body has attracted the interest of many authors. Hiemenz [11] was the first to study the two-dimensional stagnation flow using a similarity transformation to reduce the Navier–Stokes equations to nonlinear ordinary differential equations. He developed an exact solution to the Navier–Stokes equations. Merril et al. [12] investigated the large time (final state flow) solutions for unsteady mixed convection boundary layer flow near a stagnation point on a vertical surface embedded in a Darcian fluid-saturated porous medium.

Crane [13] was the first to study the problem of steady two-dimensional boundary layer flow of an incompressible viscous fluid caused by a stretching plate whose velocity varies linearly with the distance from a fixed point on the sheet. The combination of both stagnation flow and stretching surface was considered by Mahapatra and Gupta [14,15]. The flow over a shrinking sheet was investigated by Miklavčič and Wang [16]. For this flow configuration, the sheet is shrunk toward a slot and the flow is quite different from the stretching case. It is also shown that mass suction is required to maintain the flow over the shrinking sheet. The flow induced by a shrinking sheet with constant or power-law velocity distribution was investigated recently by Fang [17] and Fang et al. [18]. Wang [19] studies the stagnation flow towards a shrinking sheet and found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. The flow over an unsteady shrinking sheet was studied by Fang et al. [20] and the solution is an exact solution of the unsteady Navier–Stokes equations. This shrinking sheet problem was extended to a second grade fluid [21], and MHD rotating flow of a viscous fluid [22].

The objective of this paper is to investigate the heat transfer characteristics caused by a shrinking sheet immersed in a fluid-saturated porous medium. The results for the skin friction coefficient, local Nusselt number, velocity profiles as well as the temperature profiles are obtained and discussed for different values of the governing parameters. We restrict our study to unit Prandtl number, taking $Pr = 1$. We expect our results are qualitatively similar with other values of Pr of $O(1)$.

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**Nomenclature**

| | |
|---|---|
| $a$, $b$, $c$ | constants |
| $C_f$ | skin friction coefficient |
| $f$ | dimensionless stream function |
| $k$ | thermal conductivity |
| $K$ | permeability parameter |
| $K_1$ | permeability of the porous medium |
| $Nu_x$ | local Nusselt number |
| $Pe_x$ | local Péclet number |
| Pr | Prandtl number |
| $q_w$ | surface heat flux |
| $\mathrm{Re}_x$ | local Reynolds number |
| $T$ | fluid temperature |
| $T_w$ | surface temperature |
| $T_\infty$ | ambient temperature |
| $u$, $v$ | velocity components along the $x$ and $y$ directions, respectively |
| $\mu_e$ | velocity of the external flow |
| $\mu_w$ | velocity of the stretching surface |
| $x$, $y$ | Cartesian coordinates along the surface and normal to it, respectively |

*Greek letters*

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\beta$ | thermal expansion coefficient |
| $\eta$ | similarity variable |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\theta$ | dimensionless temperature |
| $\rho$ | fluid density |
| $\tau_w$ | surface shear stress |
| $\psi$ | stream function |

*Subscripts*

| | |
|---|---|
| $w$ | condition at the surface |
| $\infty$ | condition away from the surface |

*Superscript*

| | |
|---|---|
| $'$ | differentiation with respect to $\eta$ |

## 2. Problem formulation

Consider a steady stagnation point flow over a shrinking sheet which is embedded in a porous medium as shown in Fig. 1. The Cartesian coordinates $x$ and $y$ are taken with the origin O at the stagnation point, and are defined such that the $x$-axis is measured along the stretching/shrinking sheet and the $y$-axis is measured normal to it. It is assumed that the velocity of the external flow is given by $u_e(x) = a\,x$, where $a>0$ is the strength of the stagnation flow and the surface temperature $T_w$ is a constant. It is also assumed that the velocity of the stretching/shrinking sheet is given by $u_w(x) = b\,x$, where $b$ is the stretching rate, with $b>0$ and $b<0$ are for stretching and shrinking cases, respectively. The boundary layer equations in a porous medium are given by [23]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\nu}{K_1}(u_e - u), \tag{2}$$

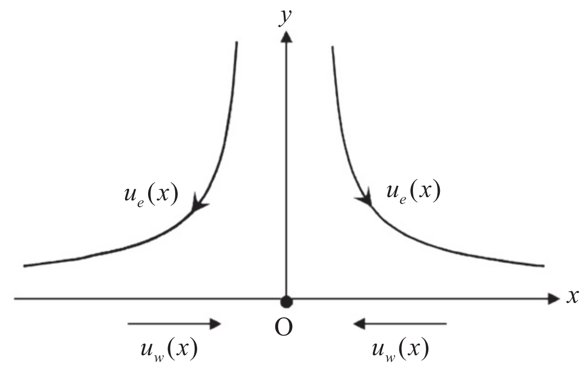


**Fig. 1.** Physical model of two-dimensional stagnation point flow over a shrinking sheet.

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &u = u_w(x) = bx, \quad T = T_w \quad \text{at} \quad y = 0, \\ &u = u_e(x) = ax, \quad T = T_\infty \quad \text{as} \quad y\to\infty, \end{aligned} \tag{4}$$

where $u$ and $v$ are the velocity components along the $x$- and $y$-axes, respectively, $T$ is the fluid temperature and the other physical quantities are defined in the Nomenclature.

To obtain similarity solutions for the system of Eqs. (1)–(4), we introduce the following similarity variables (see Cheng [24] or Lai and Kulacki [25])

$$\eta = \left(\frac{u_e x}{\alpha}\right)^{1/2}\frac{y}{x}, \quad \psi = (\alpha x u_e)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{5}$$

where $\psi$ is the stream function defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, which identically satisfy Eq. (1). Using the non-dimensional variables in Eq. (5), Eqs. (2) and (3) reduce to the following ordinary differential equations

$$\mathrm{Pr} f''' + f f'' - f'^2 + 1 + K(1 - f') = 0, \tag{6}$$

$$\theta'' + f\theta' = 0, \tag{7}$$

subject to the boundary conditions

$$\begin{aligned} &f(0) = 0, \quad f'(0) = b/a = c, \quad \theta(0) = 1, \\ &f'(\eta)\to 1, \quad \theta(\infty)\to 0 \quad \text{as} \quad \eta\to\infty, \end{aligned} \tag{8}$$

where primes denote differentiation with respect to $\eta$, $\mathrm{Pr} = \nu/\alpha$ is the Prandtl number and $K = \nu/(aK_1)$ is the permeability parameter. It is worth mentioning that $c>0$ and $c<0$ correspond to stretching and shrinking sheets, respectively, while $c = 0$ is the planar stagnation flow towards a stationary sheet. Moreover, $c = 1$ corresponds to the flow with no boundary layer ($u_w = u_e$).

The physical quantities of interest are the skin friction coefficient $C_f$ and the local Nusselt number $Nu_x$, which are defined as

$$C_f = \frac{\tau_w}{\rho u_e^2/2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \tag{9}$$

where the surface shear stress $\tau_w$ and the surface heat flux $q_w$ are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{10}$$