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Two new differential equations of turbulent dissipation rate and apparent viscosity for non-newtonian fluids ☆

Fabio Gori [a,*], Andrea Boghi [b]

[a] Department of Mechanical Engineering, University of Rome "Tor Vergata", 00133 Rome, Italy
[b] Energy and Technology Research Group, School of Engineering Science, University of Southampton, Southampton SO17 1BJ, United Kingdom

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ABSTRACT

A new equation for the dissipation rate of turbulent kinetic energy is derived exactly in conservative form for a Generalized Newtonian Fluid (GNF). The transport equations for mass, momentum, and turbulent kinetic energy are written along to the transport equation for the shear rate. A new transport equation for the apparent viscosity is derived assuming the viscosity as dependent only on the shear rate. The assumption is of incompressible two-dimensional GNF flow.

1. Introduction

Non-Newtonian fluids are present in several industrial applications and biological problems, like blood flow. Literature presents many theoretical solutions and numerical simulations in laminar flow, including two papers published by the first author more than 30 years ago [1,2].

Few numerical investigations dealt with turbulent flow of pseudo-plastic fluids (shear-thinning fluid) because of the lack of models with one or two point closure and, for this reason, some investigators performed DNS (Direct Numerical Simulation). Rudman and Blackburn [3] used the Spectral Element-Fourier Method (SEM) in a duct flow and compared the DNS results of a power law fluid with small consistency index and of a Herschel-Bulkley fluid with experimental data [4]. Dimitropoulos [5,6] carried out DNS for a polymeric solution using FENE-P and *Giesekus* models with spectral approximation and semi-implicit algorithm to predict the drag reduction. New results on Reynolds stresses and pressure are presented in [6], where the convergence of the pseudo-spectral algorithm is discussed. A non-refined mesh and a high artificial viscosity are introduced to stabilize the algorithm. The FENE-P model is used in [7] for a DNS one-dimensional approach to explain the phenomenon of drag reduction. A turbulent model for a non-Newtonian power law fluid is developed in [8], in analogy to the turbulent viscosity, determining the temperature distribution for soybean milk flowing inside a tubular heat exchanger.

Turbulent flow of a non-Newtonian fluid is important also in medical field. A model to predict the turbulent flow of a power-law fluid in a bio-reactor for anaerobic digestion is developed in [9] with the classical k-ε model and the power-law viscosity. The k-ε equations are derived in [10,11] for power-law and Herschel–Bulkley fluids using the apparent viscosity of a non-Newtonian fluid in the RANS equations of a Newtonian fluid, but the agreement is not good enough. The introduction of the third invariant of the rate of deformation tensor in the viscosity contributes to an increase of viscous diffusion and dissipation rate in the turbulent kinetic energy confirming the dependence of the viscosity on the second invariant of the rate of deformation tensor in a 2D flow [12]. The Generalized Newtonian Fluid (GNF) constitutive equation is applied to a *Bird-Carreau* fluid in order to derive a k–ε model for the equations of the Reynolds stresses tensor, turbulent kinetic energy and dissipation rate [13]. The viscosity is dependent on the invariants of the rate of deformation tensor, shear-rate and strain-rate. An algebraic equation is proposed to correlate the instantaneous viscosity to the dissipation rate while average viscosity and dissipation rate are correlated with a normal logarithmic probability distribution of the dissipation rate. The final equation of dissipation rate is written in non conservative form because two derivatives are present, one for the dissipation rate itself and the other for the average dynamic viscosity.

Direct Numerical Simulation of viscoelastic fluids in turbulent channel flow is carried out using the FENE-P model to find relationships between flow and fluid rheological parameters [14]. Three different regimes of drag reduction, namely low, high and medium are identified proposing mathematical expressions for the eddy viscosity in the three regimes. A procedure based on the DNS predictions of the budgets of momentum and viscoelastic shear stress is developed to evaluate the mean velocity profile. A RANS model is employed using the FENE-P constitutive relationship to describe the rheology of polymer-induced turbulent drag reduction [15]. Correlations among flow and polymer conformation variables are identified by analyzing recent DNS results of dilute polymer solutions.

The present work is aimed to derive the equation of the turbulent dissipation rate in a conservative form for an incompressible GNF in 2D (two-dimensional) flow. Viscosity is depending on shear-rate, as shown

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* Corresponding author.
E-mail address: fammannati@yahoo.com (F. Gori).

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Nomenclature

Latin

k	mean turbulent kinetic energy
k'	instantaneous turbulent kinetic energy
p	instantaneous static pressure
P	mean static pressure
p'	fluctuating static pressure
S	variable defined in Eq. (4)
S_{ij}	rate of deformation tensor
$\overline{S_{ij}}$	mean rate of deformation tensor
S'_{ij}	fluctuating rate of deformation tensor
$\$$	variable defined in Eq. (20)
T^R_{ij}	mean Reynolds stress tensor
T^μ_{ij}	mean polymeric stress tensor
$T^{R'}_{ij}$	instantaneous Reynolds stress tensor
$T^{\mu'}_{ij}$	instantaneous polymeric stress tensor
t	time
u_i	instantaneous i-velocity component
U_i	mean i-velocity component
x_i	i- coordinate

Greek

$\dot{\gamma}$	shear rate
δ_{ij}	Kronecher delta
ε	mean dissipation rate
ε'	instantaneous dissipation rate
μ_{app}	apparent viscosity
$\overline{\mu_{app}}$	mean apparent viscosity
μ'_{app}	fluctuating apparent viscosity
ρ	density
Ω_{ij}	rotation rate tensor
$\overline{\Omega_{ij}}$	mean rotation rate tensor
Ω'_{ij}	fluctuating rotation rate tensor

in [12] in a 2D flow and done in all the papers found in the literature, without an explicit statement of the relation with the shear-rate. A 2D theory of turbulence has been developed since the 70's in [16–20] despite the differences with the real 3D flow and the absence of vortex-stretching terms. It is also affirmed in [21] that it seems natural to investigate the behavior of a 2D flow, in the hope that it sheds some light on an "almost" 2D turbulence. Due to the difficulty of deriving the governing equations directly in a 3D flow, it is considered a guide to develop first a 2D turbulent model. The importance of studying a 2D model, which can be generalized in 3D, is also remarked in [12], where the authors developed an order of magnitude analysis for a 2D turbulent flow, without loss of generality.

The transport equation of ε is deduced in this work by using a completely new transport equation for the apparent viscosity and without a constitutive link between apparent viscosity and shear rate. No hypotheses are necessary about the dependence of the turbulent dissipation rate on the fluctuating part of the rate of deformation tensor, as required in [15], neither a particular statistical distribution of the average viscosity, as required in [16]. The present procedure allows obtaining a new transport equation of ε in a conservative form, more general than that obtained previously in the literature. Moreover, the conservative form of the ε equation allows avoiding the calculation of the temporal derivative of the apparent viscosity, due to the presence of a transport equation for the apparent viscosity.

The paper has the following structure. The transport equations for the average variables and the turbulent kinetic energy are derived first and then the transport equation for the shear-rate. The new differential equation for the apparent viscosity is deduced using the scalar product of the instantaneous rate of deformation tensor by itself. From this equation it is possible to derive the equation of dissipation rate in a conservative form giving a physical interpretation to the new terms. The method used in this work allows to explain each term under the classification of transport, production and dissipation terms.

2. Constitutive equation

The present 2D analysis is carried on for a Generalized Newtonian Fluid, GNF. The constitutive equation for an incompressible non-Newtonian fluid is written similarly to a Newtonian one with the apparent viscosity function of the shear-rate

$$T_{ij} = -p\delta_{ij} + 2\mu_{app}S_{ij}, \tag{1}$$

where T_{ij} is the stress tensor and p the static pressure.

The rate of deformation tensor S_{ij} is

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{2}$$

and the shear-rate $\dot{\gamma}$ is

$$\dot{\gamma} = \sqrt{2S^2_{ij}}. \tag{3}$$

Defining S as

$$S = \frac{\dot{\gamma}}{\sqrt{2}}, \tag{4}$$

the shear-rate will be treated as S from now on.

3. Conservation equations of mass, momentum and turbulent kinetic energy

The conservation equations for the instantaneous variables in a 2D flow are the followings:

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{5}$$

for the mass, and

$$\rho\frac{\partial u_i}{\partial t} + \rho u_k\frac{\partial u_i}{\partial x_k} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_k}\left(2\mu_{app}S_{ik}\right), \tag{6}$$

for the momentum.

Each instantaneous variable is decomposed into a mean and a fluctuating component

$$u_i = U_i + u'_i, \tag{7}$$

$$p = P + p', \tag{8}$$

$$\mu_{app} = \overline{\mu_{app}} + \mu'_{app}. \tag{9}$$

The mean component of the stress tensor, Eq. (1), becomes

$$\overline{T_{ij}} = -P\delta_{ij} + 2\overline{\mu_{app}}\,\overline{S_{ij}} + 2\overline{\mu'_{app}S'_{ij}}, \tag{10}$$

and the fluctuating one

$$T'_{ij} = -p'\delta_{ij} + 2\mu'_{app}\overline{S_{ij}} + 2\overline{\mu_{app}}S'_{ij} + 2\mu'_{app}S'_{ij} - 2\overline{\mu'_{app}S'_{ij}}. \tag{11}$$

The third term of the mean component, Eq. (10), is due to the viscosity fluctuations while different combinations of mean and