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Hermite-Padé approximation approach to thermal criticality for a reactive third-grade liquid in a channel with isothermal walls ☆

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Abstract

This investigation deals with the thermal criticality of a reactive third-grade liquid flowing steadily between two parallel isothermal plates. It is assumed that the reaction is exothermic under Arrhenius kinetics, neglecting the consumption of the material. Approximate solutions are constructed for the governing nonlinear boundary value problem using regular perturbation techniques together with a special type of Hermite-Padé approximants and important properties of the velocity and temperature fields including bifurcations and thermal criticality conditions are discussed.

Keywords: Channel flow; Third grade fluid; Arrhenius kinetics; Thermal criticality; Hermite-Padé; Approximants

1. Introduction

It is well known that the rheological properties of many fluids in engineering and industrial applications are not well modeled by Navier–Stokes equations due to their non-Newtonian behaviour [1,2]. In the past few decades there has been significant work on flows of non-Newtonian fluids, not only because of their non-linearity which occur in the inertial part, but also in the surface forces of the governing equations. The study of thermal criticality and heat transfer plays an important role during the handling and processing of non-Newtonian fluids [4,9]. Rajagopal [11] studied in detail the general thermodynamics, stability and uniqueness of fluids of the differential type with the fluid of third grade being a special case. For problems involving heat transfer for fluid of third grade, a complete thermodynamics analysis of the constitutive function has been performed by Fosdick and Rajagopal [4]. Similar studies with respect to non-Newtonian fluid are also reported in [10,14].

Mathematically speaking, the thermal boundary layer equation for non-Newtonian fluids constitute a nonlinear problem and the long-term behavior of the solutions in space will provide us an insight into inherently complex physical process of thermal criticality in the system. Hence, the purpose of the present work is to investigate the thermal criticality for a reactive third grade liquid flowing steadily between two parallel isothermal plates using a special type of

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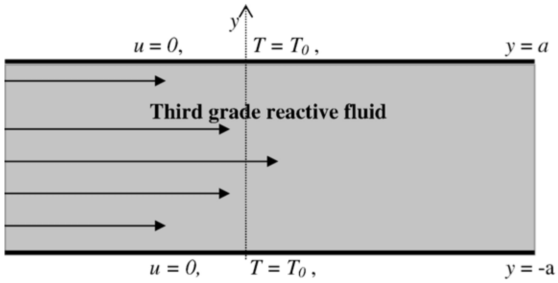

Fig. 1. Geometry of the problem.

Hermite-Padé approximants ([6–9,12,13]). The mathematical formulation of the problem is established and solved in sections two and three. In section four we introduce and apply some rudiments of Hermite-Padé approximation technique. Both numerical and graphical results are presented and discussed quantitatively with respect to various parameters embedded in the system in section five.

2. Mathematical model

Consider the steady flow of an incompressible third grade reactive fluid placed between two parallel isothermal plates (see Fig. 1). It is assumed that the fluid motion is induced by applied axial pressure gradient. We choose x-axis parallel to the plate and y-axis normal to it. For hydrodynamically and thermally developed flow, both velocity and temperature fields depend on y only. Following ([4,11,14]) and neglecting the reacting viscous fluid consumption, the one dimensional governing equations for the momentum and heat balance can be written as;

$$\mu\frac{d^2u}{dy^2}+6\beta_3\frac{d^2u}{dy^2}\left(\frac{du}{dy}\right)^2=\frac{dP}{dx}, \tag{1}$$

$$k\frac{d^2T}{dy^2}+\left(\frac{du}{dy}\right)^2\left(\mu+2\beta_3\left(\frac{du}{dy}\right)^2\right)+QC_0\,Ae^{-\frac{E}{RT}}=0, \tag{2}$$

subject to the boundary conditions:

$$\frac{du}{dy}(0)=\frac{dT}{dy}(0)=0, u(a)=0, T(a)=T_0, \tag{3}$$

where the additional Arrhenius kinetics term in energy balance equation is due to [5,9]. Here T is the absolute temperature, U the fluid characteristic velocity, T_0 the plate temperature, k the thermal conductivity of the material, Q the heat of reaction, A the rate constant, E the activation energy, R the universal gas constant, C_0 the initial concentration of the reactant species, a the channel half width, β_3 the material coefficient, P the modified pressure and μ is the fluid dynamic viscosity coefficient [1,4,10]. We introduce the following dimensionless variables into Eqs. (1)–(3);

$$\theta=\frac{E(T-T_0)}{RT_0^2},\ \ \bar{y}=\frac{y}{a},\lambda=\frac{QEAa^2C_0e^{-\frac{E}{RT_0}}}{T_0^2Rk},W=\frac{u}{UG},$$
$$m=\frac{\mu G^2U^2e^{\frac{E}{RT_0}}}{QAa^2C_0},\varepsilon=\frac{RT_0}{E},G=-\frac{a^2}{\mu U}\frac{dP}{dx},\gamma=\frac{\beta_3U^2G^2}{a^2\mu}, \tag{4}$$