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Non-Fourier heat conduction in a finite medium subjected to arbitrary non-periodic surface disturbance ☆

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Abstract

The non-Fourier heat conduction equation is solved for a finite medium under arbitrarily chosen surface disturbances by utilizing the principle of superposition along with the Fourier integral representation of arbitrary non-periodic functions. The obtained solution is such that for a given non-periodic disturbance, we just have to introduce the Fourier integral coefficients of that function to the presented general solution and perform some definite integrations, analytically if possible, but in general numerically which is a straightforward computational task.

Keywords: Non-Fourier heat conduction; Hyperbolic conduction; Finite medium; Non-periodic disturbance; Fourier integral

1. Introduction

During the past few years there has been a research concerned with departures from the classical Fourier heat conduction law. The motivation for this research was to eliminate the paradox of an infinite thermal wave speed which is in contradiction with Einstein's theory of relativity and thus provide a theory to explain the experimental data on 'second sound' in liquid and solid helium at low temperatures [1,2]. In addition to low temperature applications, non-Fourier theories have attracted more attention in engineering sciences, because of their applications in high heat flux conduction, short time behavior as found, for example, in laser-material interaction, etc.

The classical Fourier conduction law relates the heat flux vector $\mathbf{q}$ to the temperature gradient $\nabla\theta$, by the relation

$$\mathbf{q} = -\lambda\nabla\theta, \tag{1}$$

where the material constant λ is the thermal conductivity. Eq. (1) along with the conservation of energy gives the classical parabolic heat equation

$$a\Delta\theta = \frac{\partial\theta}{\partial t}, \tag{2}$$

where $a=\lambda/\rho c$, ρ, c and Δ are thermal diffusivity, mass density, specific heat capacity and Laplacian, respectively. Eq. (2) yields temperature solutions which imply an infinite speed of heat propagation.

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Nomenclature

a	thermal diffusivity, m^2/s
c	specific heat capacity, J/kg K
Fo	Fourier number, at/L^2
Fo_1	Fourier number based on frequency, $a/\omega L^2$
$i(t)$	normalized Gaussian surface disturbance defined by Eq. (16a)
$I(t)$	Gaussian surface disturbance defined by Eq. (16a)
$j(t)$	normalized non-Gaussian surface disturbance defined by Eq. (16b)
$J(t)$	non-Gaussian surface disturbance defined by Eq. (16b)
L	thickness of medium, m
q	surface disturbance, W/m^2
$\overline{q}(\omega)$	Fourier's integral coefficient defined by Eq. (11)
$\mathbf{q}$	heat flux vector, W/m^2
Q	heat source term, K/s
t	time, s
t_p	characteristic of Gaussian and non-Gaussian surface disturbances
V	dimensionless temperature defined by Eq. (7)
V_ω	dimensionless temperature corresponds to a special value of ω
Ve	Vernotte number, $\sqrt{a\tau_0/L}$
x	spatial variable, m
X	dimensionless spatial variable, x/L

Greek symbols

β,β_1	auxiliary variable defined by Eq. (9b)
λ	thermal conductivity, W/m K
θ	temperature, K
θ_ω	temperature corresponds to a special value of ω
ρ	mass density, kg/m^3
τ_0	relaxation time, s
$\xi_\pm,\zeta_\pm$	auxiliary variables defined by Eq. (9a)
ω	frequency in Fourier space, s^{-1}

In order to eliminate this paradox, Cattaneo [3] and Vernotte [4] independently postulated a time-dependent relaxation model for the heat flux in solids:

$$\mathbf{q}+\tau_0\frac{\partial\mathbf{q}}{\partial t}=-\lambda\nabla\theta, \tag{3}$$

where τ_0 is the so-called relaxation time (a non-negative constant). If the relaxation time $\tau_0=0$, the heat flux law of the non-Fourier model defined by Eq. (3) reduces to the classical Fourier's model for heat conduction, i.e., Eq. (1). However, this model was first proposed by Maxwell [5]. Inserting Eq. (3) into the energy conservation equation, the hyperbolic heat transport equation, including source term, takes the form

$$a\Delta\theta=\frac{\partial\theta}{\partial t}+\tau_0\frac{\partial^2\theta}{\partial t^2}+Q(\mathbf{x},t), \tag{4}$$

where Q is the source term and $\sqrt{a/\tau_0}$ denotes the propagation speed of temperature wave.

Various solution schemes for Eq. (4) with different initial and boundary conditions can be browsed in literature. Baumeister and Hamill [6] have given the solution for a semi-infinite problem. Sadd and Cha [7] have solved Eq. (4) for axisymmetric conduction in cylindrically bounded domains. Several solutions for finite medium have been given in