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The onset of ferroconvection in a horizontal ferrofluid saturated porous layer heated from below and cooled from above with constant heat flux subject to MFD viscosity ☆

C.E. Nanjundappa [a], I.S. Shivakumara [b,*], M. Ravisha [c]

[a] Department of Mathematics, Dr. Ambedkar Institute of Technology, Bangalore, 560 056, India
[b] UGC — Centre for Advanced Studies in Fluid Mechanics, Department of Mathematics, Bangalore University, Bangalore, 560 001, India
[c] Smt. Rukmini Shedthi Memorial National Government First Grade College, Department of Mathematics, Barkur, 576 210, Udupi District, India

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ABSTRACT

The effect of magnetic field dependent (MFD) viscosity on the onset of ferroconvection in a ferrofluid saturated horizontal porous layer is investigated theoretically. The bounding surfaces of the porous medium are considered to be either rigid-ferromagnetic or stress free with constant heat flux conditions. The resulting eigenvalue problem is solved numerically using the Galerkin technique and also analytically by regular perturbation technique. It is found that increase in porous parameter, MFD viscosity parameter and decrease in the magnetic number is to delay the onset of ferroconvection, while the nonlinearity of fluid magnetization has no influence on the stability of the system.

1. Introduction

In the 1960s, scientists from the National Aeronautics and Space Administration (NASA) research centre were investigating methods for controlling liquids in space. They developed what are now called ferrofluids, which are colloidal suspensions of magnetic nanoparticles in a carrier fluid such as water, hydrocarbon (mineral oil or kerosene) or fluorocarbon. The nanoparticles typically have sizes of about 100 Å or 10 nm and they are coated with surfactants in order to prevent the coagulation. Ferrofluids respond to an external magnetic field and this enables to control the location of the fluid through the application of a magnetic field. Ferrofluids possess a wide variety of potential applications in many fields ranging from mechanical engineering to biomedical applications. An authoritative introduction to this fascinating subject is provided in [1–3]. Thus, ferrofluids have received much attention in the scientific community.

The magnetization of ferrofluids depends on the magnetic field, temperature, and density. Hence, any variations of these quantities induce change of body force distribution in the fluid and eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field. There have been numerous studies on thermal convection in a ferrofluid layer called ferroconvection analogous to Rayleigh–Benard convection in ordinary viscous fluids. The theory of thermal convective instability in a ferrofluid layer began with Finlayson [4] and extensively continued over the years ([5–9]). Recently, Nanjundappa and Shivakumara [10] have investigated a variety of velocity and temperature boundary conditions on the onset of ferroconvection in an initially quiescent ferrofluid layer in the presence of a uniform magnetic field.

Thermal convection of ferrofluids saturating a porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageable liquids into particular zones for subsequent imaging etc. Rosensweig et al. [11] have studied experimentally the penetration of ferrofluids in the Hele–Shaw cell. The stability of the magnetic fluid penetration through a porous medium in high uniform magnetic field oblique to the interface is studied by Zahn and Rosensweig [12]. The thermal convection of a ferrofluid saturating a porous medium in the presence of a vertical magnetic field is studied by Vaidyanathan et al. [13]. The laboratory-scale experimental results of the behavior of ferrofluids in porous media consisting of sands and sediments are presented by Borglin et al. [14]. Recently, Shivakumara et al. [15] have investigated the criterion for the onset of convection in a horizontal ferrofluid saturated porous layer for various types of velocity and temperature boundary conditions.

Thermal convection in ferromagnetic fluids is gaining much importance due to their astounding physical properties. One such property is the viscosity of the ferromagnetic fluid. Fluids with ferromagnetic properties may be formed by colloidal suspension of solid magnetic particles such as magnetite in a parent liquid. Viscosity of the fluid in a magnetic field is predicted by dimensional analysis to be a function of the ratio of hydrodynamic stress to magnetic stress (Rosensweig et al. [16]). The effect of homogeneous magnetic field on the viscosity of a ferrofluid with solid particles possessing intrinsic magnetic moments was investigated by Shliomis [17]. The effect of magnetic field dependent (MFD) viscosity on ferroconvection in an

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* Corresponding author.
E-mail addresses: cenanju@hotmail.com (C.E. Nanjundappa), shivakumarais@gmail.com (I.S. Shivakumara), pmravisha@yahoo.co.in (M. Ravisha).

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anisotropic porous medium has been studied by Ramanathan and Suresh [18]. Kaloni and Lou [19] have investigated theoretically the convective instability problem in a thin horizontal layer of magnetic fluid heated from below under alternating magnetic field by considering the quasi stationary model with internal rotation and vortex viscosity. Recently, Sunil et al. [20] have studied theoretically the effect of magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid layer with or without dust particles.

The present study deals with the onset of ferroconvection in a horizontal saturated porous layer by employing a non-Darcian model for different combinations of velocity boundary conditions with constant heat flux subject to MFD viscosity. The resulting eigenvalue problem is solved numerically using the Galerkin technique and analytically by the regular perturbation technique for rigid–rigid, free–free, and lower boundary rigid and upper boundary free boundary combinations.

To achieve the above objectives, the paper is organized as follows. Section 2 is devoted to mathematical formulation. The method of solution is discussed in Section 3. In Section 4, the numerical results are discussed and some important conclusions follow in Section 5.

2. Mathematical formulation

The system considered is initially quiescent ferrofluid saturated horizontal porous layer of characteristic thickness d in the presence of an applied magnetic field H_0 in the vertical direction. The lower and the upper boundaries of the porous layer are maintained at constant temperature T_0 and $T_1(<T_0)$ respectively, and thus constant temperature difference $\Delta T(=T_1-T_0)$ is maintained between boundaries. A Cartesian co-ordinate system (x, y, z) is used with the origin at the bottom of the porous layer and z-axis is directed vertically upward. The gravitational force $(0,0,-g)$acts in the negative z-direction. The flow in the porous medium is described by the Brinkman–Lapwood extended Darcy momentum equation containing viscous force $2\nabla\cdot(\eta \underline{D})$, where $\underline{\underline{D}} = [\nabla\vec{q} + (\nabla\vec{q})^T]/2$ is the rate of strain tensor and $\vec{q} = \overline{(u, v, w)}$ is the velocity vector. The fluid is assumed to be incompressible having variable viscosity, given by $\eta = \eta_0(1 + \vec{\delta}\cdot\vec{B})$, where $\vec{\delta}$ is the variation coefficient of magnetic field dependent viscosity, η_0 is taken as viscosity of the fluid when the applied magnetic field is absent and $\vec{B} = (B_x, B_y, B_z)$ is the magnetic induction. Experimentally, it has been demonstrated that the magnetic viscosity has got exponential variation with respect to magnetic field (Rosensweig [21]). As a first approximation for small field variation, linear variation of magnetic viscosity has been used.

It is clear that there exists the following solution for the basic state:

$$\vec{q}_b = 0, \quad p_b(z) = p_0 - \rho_0 g z - \rho_0 \alpha_t g \beta z^2/2 - \mu_0 M_0 K \beta z/(1+\chi) \quad (1)$$
$$-\mu_0 K^2 \beta^2 z^2 / 2(1+\chi)^2, \quad T_b(z) = T_0 - \beta z, \quad \vec{H}_b(z)$$
$$= [H_0 - K\beta z/(1+\chi)]\hat{k}, \quad \vec{M}_b(z) = [M_0 + K\beta z/(1+\chi)]\hat{k}.$$

Here, $\vec{M}$ is the magnetization, $\vec{H}$ the magnetic intensity of the fluid, p the pressure, ρ_o the density at $T=T_0$, μ_o the magnetic permeability of vacuum, α_t the thermal expansion coefficient, $\hat{k}$ the unit vector in the z-direction, β temperature gradient, $\chi = (\partial M/\partial H)_{H_0, T_0}$ the magnetic susceptibility, $K = -(\partial M/\partial T)_{H_0, T_0}$ the pyromagnetic coefficient and $M_0 = M(H_0, T_0)$.

To investigate the conditions under which the quiescent solution is stable against small disturbances, we consider a perturbed state such that

$$\vec{q} = \vec{q}', \quad p = p_b(z) + p', \quad \eta = \eta_b(z) + \eta', \quad T = T_b(z) \quad (2)$$
$$+ T', \vec{H} = \vec{H}_b(z) + \vec{H}', \quad \vec{M} = \vec{M}_b(z) + \vec{M}'$$

where $\vec{q}'$, p', η', T′, $\vec{H}'$, and $\vec{M}'$, are perturbed variables and are assumed to be small.

Following the standard linear stability analysis procedure [4] and noting that the principle of exchange of stability is valid, the resulting dimensionless equations are then found to be

$$(1+\delta)\left[\left(D^2-a^2\right)-\sigma^2\right]\left(D^2-a^2\right)W = -a^2 R[M_1 D\Phi - (1+M_1)\Theta] \quad (3)$$

$$\left(D^2-a^2\right)\Theta = -(1-M_2 A)W \quad (4)$$

$$\left(D^2-a^2 M_3\right)\Phi - D\Theta = 0. \quad (5)$$

Here $D = d/dz$ is the differential operator, $a = \sqrt{\ell^2+m^2}$ the overall horizontal wave number, W the amplitude of vertical component of velocity, Θ the amplitude of temperature, Φ the amplitude of magnetic potential, $R = \alpha_t g \beta d^4/\nu\kappa A$ the thermal Raleigh number, $M_1 = \mu_0 K^2 \beta/(1+\chi)\alpha_t \rho_0 g$ the magnetic number, $M_2 = \mu_0 T_0 K^2/(1+\chi)(\rho_0 C)_1$ the magnetic parameter, $M_3 = (1+M_o/H_o)/(1+\chi)$ the measure of nonlinearity of magnetization, $\sigma = d/\sqrt{k}$ the porous parameter and $A = (\rho_0 C)_1/(\rho_0 C)_2$ the ratio of heat capacities. The typical value of M_2 for magnetic fluids with different carrier liquids turns out to be of the order of 10^{-6} and hence its effect is neglected as compared to unity.

The boundaries are considered to be either rigid-ferromagnetic or stress free with constant heat flux conditions at the boundaries. Thus, on the rigid-ferromagnetic boundary, $W = DW = \Phi = D\Theta = 0$ and on the stress free boundary, $W = D^2W = D\Phi = D\Theta = 0$.

3. Method of solution

Eqs. (3)–(4) together with the corresponding boundary conditions constitute an eigenvalue problem with R as an eigenvalue. The eigenvalue problem is solved numerically using the Galerkin technique as well as analytically using a regular perturbation technique and the results so obtained are compared to know the accuracy of the methods employed.

3.1. Solution by Galerkin technique

The Galerkin method is used to solve the eigenvalue problem as explained in the book by Finlayson [22]. In this method, the test (weighted) functions are the same as the base (trial) functions. Accordingly, W, Θ and Φ are written as

$$W = \sum_{i=1}^{n} A_i W_i(z), \quad \Theta(z) = \sum_{i=1}^{n} C_i \Theta_i(z), \quad \Phi(z) = \sum_{i=1}^{n} D_i \Phi_i(z) \quad (6)$$

where A_i, C_i and D_i are unknown constants to be determined. The base functions $W_i(z)$, $\Theta_i(z)$ and $\Phi_i(z)$ are generally chosen such that they satisfy the corresponding boundary conditions but not the differential equations. For rigid–rigid, rigid–free and free–free boundaries, they are chosen respectively as

$$W_i = \left(z^4 - 2z^3 + z^2\right)T^*_{i-1}, \quad \Theta_i = z^2(1-2z/3)T^*_{i-1}, \quad \Phi_i = \left(z^2 - z\right)(z-2)T^*_{i-1} \quad (7)$$

$$W_i = \left(2z^4 - 5z^3 + 3z^2\right)T^*_{i-1}, \quad \Theta_i = z^2(1-2z/3)T^*_{i-1}, \quad \Phi_i = z^2(1-2z/3)T^*_{i-1} \quad (8)$$

$$W_i = \left(z^4 - 2z^3 + z\right)T^*_{i-1}, \quad \Theta_i = z^2(1-2z/3)T^*_{i-1}, \quad \Phi_i = z^2(1-2z/3)T^*_{i-1} \quad (9)$$

where, T^*_i's are the modified Chebyshev polynomials. The above trial functions satisfy all the boundary conditions. Multiplying Eq. (3)