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Inverse method in simultaneously estimate internal heat generation and root temperature of the T-shaped fin☆

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ABSTRACT

This study employed the finite element method and rearranged the matrix to establish an inverse operation without iteration, in order to simultaneously estimate the internal heat generation and root temperature of the T-shaped fin. The results of numerical validation showed that the optimized the T-shaped fin has a good effect because of better efficiency of heat transfer. Regardless of the configurations, the estimation of internal heat generation is obviously affected by the measurement error. Nevertheless, the measurement of temperature in future time can reduce sensitivity of internal heat generation and root temperature to the measurement error.

1. Introduction

Current studies on the optimal configuration T-shaped fin suggested that the optimal configuration is when the ratio of sectional area to total area is constant [1–3]. However, there is no discussion on the T-shaped fin in studies on the inverse operation [4–7]. It is mainly because the inverse analysis of the conventional fin only focuses on simple geometric configuration or converts the fins to a one-dimensional heat transfer mode for analysis. Hsu et al. [8] suggested that the estimation of heat flux and boundary temperature at the same time is difficult when using the same method in a multidimensional inverse operation of heat transfer. In addition, to simultaneously estimate of root temperature and internal heat generation of the fin under nuclear radiation (Razelos and Satyaprakash [9]) is more difficult. Thus, this study aimed to establish an inverse operation for an irregular geometric configuration to determine the root temperature of the T-shaped fin and internal heat generation through rearrangement of discrete matrix without iteration. Additionally, the future time is added to compare the estimated results of the optimal T-shaped fin and square fin to increase stability of estimation.

2. Method and mode definition

2.1. Problem statement and mathematical derivation

Fig. 1 shows the two-dimensional T-shaped fin, and the area fraction of it is defined as

$$\varphi = \frac{A_f}{A} = \frac{2L_0t_0 + t_1L_1}{2L_0L_1} \tag{1}$$

where, areas of A_f and A are the sectional area and frontal area of the fin, respectively. If the characteristic length is $A^{1/2}$, the non-dimensional transient heat transfer equation with the initial and boundary temperature is as follows [1–3]:

$$\frac{\partial\theta}{\partial\tau} = \nabla^2\theta + q(\tau), \quad (x,y)\in V \tag{2.1}$$

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Nomenclature

{ }	row vector
[ ]	matrix
A	area
N	estimated number
$q(\tau)$	unknown internal heat generation
r	number of future time steps
$[u^i]$	column vector, a unit vector with a unit at i-component
x,y	coordinates
Greek symbols	
σ	standard deviation of measurement error
λ	weight of time step
ϕ	unknown value
θ	non-dimensional temperature
$\theta_b(\tau)$	unknown root temperature
τ	non-dimensional time
ε	absolute average error, $\varepsilon = \frac{1}{N}\sum_{j=1}^{N} \lvert\varphi_{est} - \varphi_{exact}\rvert$
Subscripts	
est	estimate
m,n	time index
$\overline{m}$	$m-2(1-\lambda)$
mea	measured
p	numbers of measure point
Superscripts	
i	measure point number

$$\theta = \theta_b(\tau) \quad \text{at} \quad y = 0 \tag{2.2}$$

$$\frac{\partial\theta}{\partial x} = 0 \quad \text{at} \quad x = 0 \tag{2.3}$$

$$\frac{\partial\theta}{\partial \overline{n}} = -Bi \;\; \theta \text{ at the other surfaces} \tag{2.4}$$

where, $\theta(\tau,x,y)$ is the distribution of the non-dimensional temperature field; $\theta_b(\tau)$ and $q(\tau)$ are the root temperature and internal heat generation of the non-dimensional fin, respectively; Bi number used by Bejan and Almogbel [1] is defined as

$$Bi = hA^{1/2} / k \tag{3}$$

The linear least-squares error method (Yang and Chen [10]) uses an inverse matrix for inverse operation. In this calculation process, discretization is first carried out for inverse problems to establish a linear inverse model, and the linear least-squares error method is used to solve the inverse model. The unknown state can be expressed through rearrangement of the matrix of the inverse problems to avoid iterative process and choice initial guess value, so the advantage of the one-step solution of optimal estimation is obvious. However, the array arrangement in the previous methods lacks simple and definite mathematical expression, and is only used for analysis of simple configuration (Chen et al. [4] and Yang [6]). Thus, in this study, finite element method instead of spatial discretization is used for the solution of the spatial

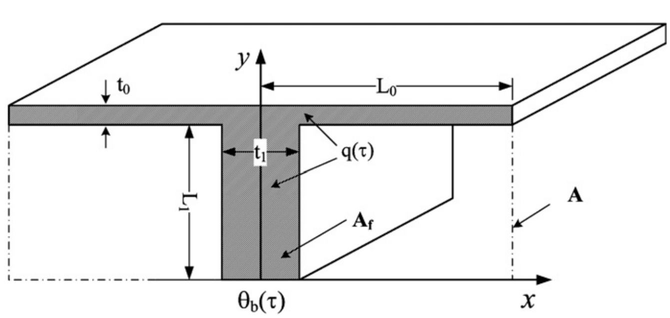

Fig. 1. Geometry configuration. (a) Rectangular figure ($L_1/L_0 = 0.5$ and $t_1/t_0 = 1$). (b) Optimal configuration (Bejan [1], $L_1/L_0 = 0.07$and $t_1/t_0 = 4$).