International Communications in Heat and Mass Transfer 38 (2011) 324–328

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

# Thin film flow over a heated nonlinear stretching sheet in presence of uniform transverse magnetic field ☆

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ARTICLE INFO

Available online 10 December 2010

*Keywords:*
Thin liquid film
Stretching sheet
Free surface flow
Heat transfer
Magnetic field

ABSTRACT

The unsteady flow of a thin viscous liquid film over a heated horizontal stretching surface permitted by uniform transverse magnetic field is studied by considering the stretching velocity and the temperature distribution in their general functional forms. An evolution equation for the film thickness is derived using long-wave approximation theory of thin liquid film and this nonlinear PDE is solved numerically for some representative values of non-dimensional parameters. It is observed that the magnetic field resists the film thinning process for all types of velocity and temperature distribution. But thermocapillarity enhances the film thinning rate even in presence of magnetic field. Further, effect of Marangoni and Prandtl numbers are explored in presence of magnetic field. Physical explanations are provided to understand the different effects on film thinning.



## 1. Introduction

The flow and heat transfer phenomena of a viscous liquid over a stretching surface has applications in a number of technological processes such as metal and polymer extrusion, continuous casting, drawing of plastic sheets, cable coating etc. In these processes, the quality of the final product very much depends on the rate of heat and mass transfer of the system. Crane [1] first studied the steady two-dimensional boundary layer flow due to the stretching of a flat elastic sheet. Due to its practical applications, the stretching sheet flow problem has attracted several researchers for the last three decades and is extensively studied to understand the same, along with either the sole effects of rotation, heat and mass transfer, chemical reaction, MHD, suction/injection, different non-Newtonian fluid or different possible combinations of these above effects ([2–14]). Needless to say that in all these studies, the boundary layer equation is considered and the boundary conditions are prescribed at the sheet and on the fluid outside the boundary layer tends to infinity. The imposition of similarity transformation reduces the system to a set of ODEs, which are then solved either analytically or numerically. To the best of our knowledge, the study of unsteady flow due to the stretching of a sheet has not yet received adequate attention when the film and the boundary layer thickness coincide, despite the extensive research made on this flow problem since the last three decades. A few research works have been carried out by considering unsteady stretching of a thin liquid film placed over the sheet. In these studies also a special type of transformation is used to express the boundary-layer equations into their similarity form and solved either numerically or analytically (Wang [15], Andersson et al. [16], Chen [17] and Dandapat et al. ([18,19]) and many others). In these investigations crude assumptions were that the initial liquid distribution over the stretching sheet is uniform and the film height varies only with time but not along the space. In general film thickness varies with time and space during stretching. Recently, Dandapat et al. [20] and Dandapat and Maity [21] have studied the development of thin liquid film over a stretching sheet with non-planar film free surface at the onset of stretching. In these studies they have assumed that the boundary layer covers the entire depth of the fluid and the Navier-Stokes equations are solved analytically using the matched asymptotic method and the method of characteristic to predict the variation of film thickness with space and time. But the tacit assumption in these studies was that the sheet is stretched linearly. Later, nonlinear stretching was studied by Santra and Dandapat ([22,23]) for non-planar film surface and solved numerically. In [23], they have found that the thermocapillary effects are responsible in shaping the film thickness. Further the thermocapillary effects are more pronounced for lower values of Prandtl number and Biot number.

In the present article, we have tried to explore the effect of transverse magnetic field and thermocapillarity on film thinning process and temperature development across the interface for linear/nonlinear stretching velocity and temperature distribution in stretching sheet. It is to be mentioned here that the development of conducting liquid film has equally important applications in the industry. The remaining parts of the paper are arranged as follows. In the next section, governing equations are expressed into their dimensionless form and are solved by using long wave theory to

☆ Communicated by A.R. Balalerishnan.

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doi:10.1016/j.icheatmasstransfer.2010.11.009

**Nomenclature**

| | |
|---|---|
| $B$ | Biot number, $\alpha h_0/k$ |
| $\mathbf{B}_0$ | magnetic field ($\text{kg}^{1/2}\ \text{m}^{-1/2}\ \text{s}^{-1}$) |
| $Ca$ | Capillary number, $\varepsilon^2/S$ |
| $c_p$ | specific heat at constant pressure ($\text{J kg}^{-1}\ \text{K}^{-1}$) |
| $Fr$ | modified Froude number, $gh_0^3/\nu^2$ |
| $\mathbf{g}$ | gravitational acceleration ($\text{m s}^{-2}$) |
| $g$ | vertical component of gravitational acceleration ($\text{m s}^{-2}$) |
| $h$ | film thickness (m) |
| $h_0$ | characteristic length scale in the vertical direction (m) |
| $k$ | thermal conductivity ($\text{W m}^{-1}\ K^{-1}$) |
| $L$ | characteristic length scale in the horizontal direction (m) |
| $M$ | Hartmann number, $\sqrt{\dfrac{\lambda \mathbf{B}_0^2 h_0 L}{\rho\nu}}$ |
| $M_w$ | effective Marangoni number, $h_0\gamma(T_{S_0}-T_a)/\rho\nu^2$ |
| $\mathbf{n}$ | unit normal vector on the interface |
| $Pr$ | Prandtl number, $\rho c_p \nu/k$ |
| $p$ | pressure ($\text{kg m}^{-1}\ \text{s}^{-2}$) |
| $S$ | surface tension parameter, $\varepsilon^2\sigma_a h_0/\rho\nu^2$ |
| $\mathbf{t}$ | tangential vector on the interface |
| $t$ | time (s) |
| $T$ | temperature (K) |
| $U$ | sheet velocity ($\text{m s}^{-1}$) |
| $V$ | velocity of the fluid ($\text{m s}^{-1}$) |
| $u$ | horizontal velocity component ($\text{m s}^{-1}$) |
| $w$ | vertical velocity component ($\text{m s}^{-1}$) |
| $x$ | horizontal coordinate (m) |
| $z$ | vertical coordinate (m) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | rate of heat transport ($\text{W m}^{-2}\ \text{K}^{-1}$) |
| $\varepsilon$ | aspect ratio, $h_0/L$ |
| $\gamma$ | constant ($\text{K}^{-1}$) |
| $\Theta$ | dimensionless temperature of the stretching sheet |
| $\nu$ | kinametic viscosity ($\text{m}^2\ \text{s}^{-1}$) |
| $\rho$ | density ($\text{kg m}^{-3}$) |
| $\sigma$ | surface tension ($\text{kg s}^{-2}$) |
| $\tau$ | stress tensor |
| $\lambda$ | electrical conductivity ($\text{m}^{-2}\ \text{s}$) |
| $\Psi$ | stream function |

*Subscripts*

| | |
|---|---|
| $a$ | ambient |
| $s$ | sheet |

*Superscript*

| | |
|---|---|
| * | dimensionless variable (from Eq. (10) and onwards all the variables are in their dimensionless form) |

obtain the evolution equation for the film thickness. Then the evolution equation is solved by using a numerical technique and is documented in Section 3. The results and discussions are mentioned in Section 4. Finally, conclusions are drawn in Section 5.

## 2. Mathematical formulation

We consider a conducting incompressible Newtonian liquid film over a heated flat elastic sheet in presence of uniform transverse magnetic field $\mathbf{B}_0$, as shown in Fig. 1. Further we consider that a non-planar liquid film of thickness $\delta(x)$ is distributed initially over the stretching surface that is laying horizontally along the $x$-axis and the gravitational acceleration ($\mathbf{g}=(0,\ -g)$) acts vertically along the negative $z$-direction. We assume that the surface at $z=0$ starts stretching impulsively from rest with the stretching velocity $U(x)$. The liquid is supposed to be non-volatile and thin so that evaporation and buoyancy effect can be ignored. The liquid properties are taken to be constant, except that the surface tension that varies linearly with temperature $\sigma=\sigma_a(1-\gamma(T-T_a))$, where $\gamma=-\dfrac{1}{\sigma_a}\dfrac{d\sigma}{dT}$ in general, $\sigma$ decreases with an increase of temperature so that $\gamma>0$ and $T_a$ is the ambient gas temperature far from liquid–gas interface. The motion of the liquid due to stretching is governed by

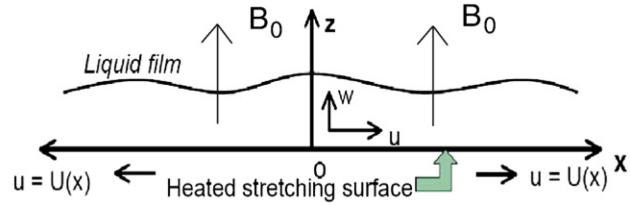


**Fig. 1.** Schematic diagram of the flow problem.

$$\nabla.\mathbf{V}=0, \tag{1}$$

$$\mathbf{V}_t+(\mathbf{V}.\nabla)\mathbf{V}=-\nabla p/\rho+\nu\nabla^2\mathbf{V}+\mathbf{g}+\frac{\lambda}{\rho}(\mathbf{V}\times\mathbf{B}_0)\times\mathbf{B}_0, \tag{2}$$

$$\rho c_p[T_t+(\mathbf{V}.\nabla)T]=k\nabla^2 T, \tag{3}$$

where $\mathbf{V}(=(u,w))$ represents the liquid velocity with $u$ and $w$ represent its components in their respective directions. $\mu$, $\rho$, $\nu$, $c_p$, $k$ and $\lambda$ represent the viscosity, density, kinematic viscosity, heat capacity at constant pressure, thermal conductivity and electrical conductivity of the liquid respectively.

The boundary conditions on the stretching sheet at $z=0$, are no-slip, no penetration and imposed sheet temperature distribution and they are represented respectively as,

$$u(t,x,0)=U(x),\quad w(t,x,0)=0,\quad T=T_S(x), \tag{4}$$

where $T_S(x)$ is the imposed sheet temperature. On the free surface at $z=h(x,t)$, the boundary conditions constitute the balance of normal and tangential stress with surface tension times of curvature and thermal stress respectively. The Newton's law of cooling and the kinematic condition also act on the free surface,

$$p_a+\mathbf{n}.\tau.\mathbf{n}=-(\nabla.\mathbf{n})\sigma, \tag{5}$$

$$\mathbf{t}.\tau.\mathbf{n}=\mathbf{t}.\nabla\sigma, \tag{6}$$

$$-k\nabla T.\mathbf{n}=\alpha(T-T_a), \tag{7}$$

$$h_t=w-uh_x, \tag{8}$$

where $\tau$ is the stress tensor for the liquid, $\mathbf{n}$ and $\mathbf{t}$ are the unit normal and tangential vectors at the interface and $\alpha$ is the rate of heat transport from the liquid to the ambient gas phase.

The governing equations and boundary conditions are expressed into their dimensionless form by using the following scaling:

$$\begin{aligned} &x=Lx^*,(z,h)=h_0(z^*,h^*),(u,U)=(\nu/h_0)(u^*,U^*),\\ &w=(\varepsilon\nu/h_0)w^*,\quad t=\left(h_0^2/\varepsilon\nu\right)t^*,\ p=\left(\rho\nu^2/\varepsilon h_0^2\right)p^*+p_a,\\ &T=T_a+\left(T_{S_0}-T_a\right)T^* \end{aligned} \tag{9}$$