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Effects of discrete isoflux heat source size and angle of inclination on natural convection heat transfer flow inside a sinusoidal corrugated enclosure ☆

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ABSTRACT

The main objective of this article is to study numerically a two-dimensional, steady and laminar viscous incompressible flow in a sinusoidal corrugated inclined enclosure. In this analysis, two vertical sinusoidal corrugated walls are maintained at a constant low temperature whereas a constant heat flux source whose length is varied from 20 to 80% of the total length of the enclosure is discretely embedded at the bottom wall. The Penalty finite element method has been used to solve the governing Navier–Stokes and energy conservation equation of the fluid medium in the enclosure in order to investigate the effects of inclination angles and discrete heat source sizes on heat transfer for different values of Grashof number. Results are presented in the form of streamline and isotherm plots. It is concluded that the average Nusselt number increases as inclination angle increases for different heat source sizes.

1. Introduction

Natural convection results when there is a fluid density gradient in a system with a density-based body force such as the gravitational force. It has been studied extensively, both experimentally and numerically, because of its various applications in engineering, such as thermal control in electronic equipments, nuclear reactors, solar collectors, and chemical vapor deposition reactors etc. Heat transfer by natural convection depends on the convection currents developed by thermal expansion of the fluid particles. Further, the shape of the heat transfer surfaces influences the development of the boundary layer. Therefore, the investigation of thermal and fluid flow behaviors for different shapes of the heat transfer surfaces is necessary to ensure the efficient performance of the various heat transfer equipments.

Several investigations have been carried out on natural convection heat transfer and fluid flow with corrugated surfaces. Chinnappa [1] carried out an experimental investigation on natural convection heat transfer from a horizontal lower hot vee corrugated plate to an upper cold flat plate. He took data for a range of Grashof numbers from 10^4 to 10^6. Randall [2] studied natural convection between a vee corrugated plate and a parallel flat plate to find the temperature gradient to estimate the local heat transfer coefficient. Local values of heat transfer coefficient were investigated over the entire vee corrugated surface area. Using control volume based finite element method, Ali and Husain [3] investigated the natural convection heat transfer and flow characteristics in a square duct of vee corrugated vertical walls. Ali and Husain [4] also investigated the effect of corrugation frequencies on natural convection heat transfer and flow characteristics in a square enclosure of vee corrugated vertical walls. This investigation showed that the overall heat transfer through the enclosure increased with the increase of corrugation for low Grashof number; but there was a reverse trend for high Grashof number. Later Ali and Ali [5] carried out a finite element analysis of laminar convection heat transfer and flow of the fluid bounded by vee corrugated vertical plate of different corrugation frequencies. Noorshahi et al. [6] studied heat transfer mechanism in an enclosure with corrugated bottom surface having uniform heat flux and flat isothermal cooled top surface and adiabatic sidewalls. Their results showed that the pseudo-conduction region was increased with the increase of the wave amplitude. Yao [7] studied theoretically the natural convection along a vertical wavy surface. He found that the local heat transfer rate was smaller than that of the flat plate case and decreased with the increase of the wave amplitude. The average Nusselt number also showed the same trend. Adjlout et al. [8] reported a numerical study of the effect of a hot wavy wall in an inclined differentially heated square cavity. Tests were performed for different inclination angles, amplitudes and Rayleigh numbers for one and three undulation. The trend of the local heat transfer was found to

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Nomenclature

g	gravitational acceleration [m/s^2]
Gr	Grashof number, $g\beta\Delta TW^3/\upsilon^2$
H	height of the enclosure [m]
J	Jacobian matrix
L	length of the heat source [m]
N_i	standard six-noded shape function
Nu	Nusselt number, Eq. (8)
p	pressure [Pa]
P	dimensionless pressure, $pW^2/\rho\upsilon^2$
Pr	Prandtl number, υ/α
q	heat flux [W/m^2]
Ra	Rayleigh number, $Gr \times Pr$
R_i	residual equations
T	temperature [K]
T_c	temperature of the cold surface [K]
u	velocity component in x-direction [m/s]
U	dimensionless velocity component in X-direction, uW/υ
v	velocity component in y-direction [m/s]
V	dimensionless velocity component in Y-direction, vW/υ
W	width of the enclosure [m]
x, y	Cartesian coordinates [m]
X, Y	dimensionless Cartesian coordinates, $(x,y)/W$

Greek symbols

Φ	inclination angle [rad]
γ	Penalty parameter
k	thermal conductivity of fluid [W/m^2 k]
α	thermal diffusivity [m^2/s]
β	coefficient of volumetric expansion [1/K]
ε	discrete heat source size ratio, L/W
θ	dimensionless temperature, $T - T_c/(qW/K)$
θ_S	local dimensionless surface temperature
ν	kinematic viscosity [m^2/s]
ρ	fluid density [kg/m^3]
Γ	dummy variable

Subscripts

c	cold wall

be wavy in nature. Due to the practical importance of flow and heat transfer in corrugated geometry many researchers have been reported results on this geometry theoretically as well as experimentally (Asako and Faghri [9]; Fabbri [10]; Goldstein and Sparrow [11]; O'Brien and Sparrow [12]; Sunden and Trollheden [13]; Xiao et al. [14]). It may also be noted that the sinusoidal wall temperature variation may produce uniform melting of materials such as glass (Sarris et al. [15]).

In this investigation, a natural convection problem has been solved for sinusoidal corrugation geometry and air has been taken as the working fluid. The corrugation geometry and the coordinate systems are shown in Fig. 1. It consists of a sinusoidal corrugated enclosure of dimensions, $W \times H$. In this work, two side walls are maintained at a constant temperature T_c, a constant flux heat source, q is discretely embedded at the bottom wall, and the remaining parts of the bottom surface and the upper wall are considered to be adiabatic. The enclosure has the same height and width, $H = W$ with single corrugation frequency and the corrugation amplitude has been fixed at 10% of the enclosure length. The ratio of the heating element to the enclosure width, $\varepsilon = L/W$ is varied from 0.2 to 0.8 and inclination angle of the enclosure, Φ is varied from 0° to 45°. The Grashof number, Gr is varied from 10^3 to 10^6 and Prandtl number, Pr is taken as 0.71.

2. Mathematical model

Natural convection is governed by the differential equations expressing conservation of mass, momentum and energy. In the present study, we consider a steady two-dimensional laminar flow of a viscous incompressible fluid. The viscous dissipation term in the energy equation is neglected. The Boussinesq approximation is invoked for the fluid properties to relate density changes to temperature change and to couple in this way the temperature field to the flow field. Then the governing equations for steady natural convection can be expressed in the dimensionless form as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + Gr\,\theta\sin\Phi \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Gr\,\theta\cos\Phi \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

where U and V are the velocity components in the X and Y directions, respectively, θ is the temperature, P is the pressure, Φ is the inclination angle of the enclosure and Gr and Pr are the Grashof number and the Prandtl number, respectively, and they are defined as:

$$Gr = \frac{g\beta\Delta TW^3}{\upsilon^2} \quad \text{and} \quad Pr = \frac{\upsilon}{\alpha}. \tag{5}$$

The dimensionless parameters in the equations above are defined as follows:

$$X = \frac{x}{W}, Y = \frac{y}{W}, U = \frac{uW}{\upsilon}, V = \frac{vW}{\upsilon}, P = \frac{pW^2}{\rho\upsilon^2}, \theta = \frac{T - T_c}{\Delta T} \quad \text{and} \quad \Delta T = \frac{qW}{k} \tag{6}$$

where ρ, β, υ, α and g are the fluid density, coefficient of volumetric expansion, kinematic viscosity, thermal diffusivity, and gravitational acceleration, respectively. The corresponding boundary conditions for

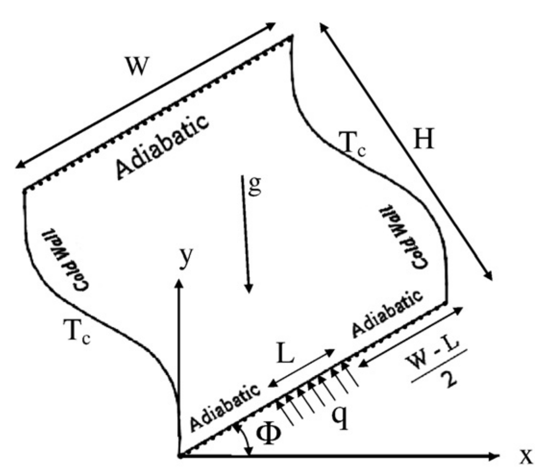

Fig. 1. Schematic diagram of the physical domain.