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On a mixed nonstationary problem of heat conduction for a half-space ☆

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Abstract

The paper deals with some mixed nonstationary problem of heat conduction for a homogeneous rigid half-space. The body is assumed to be initially at zero temperature, the boundary plane is kept at a known temperature $f(t)$ for time $0 \leq t \leq t_0$ and is insulated for $t > t_0$. The exact solution is obtained by using the integral Fourier method. Some special case is solved in details.

Keywords: Temperature; Rigid homogeneous half-space; Heat conduction

1. Introduction

Many important problems related to nonstationary heat conduction in structures are being presented for engineering applications. One of them is thermography which is a technique based on local thermal infrared emission of a sample which is monitored (see, for instance Refs. [1–3]). The interaction of thermal waves with discontinuities of materials is used for nondestructive testing of ceramics, coatings, metals, polymers. The thermography and thermal sensitivity analysis [4] require some solutions of initial-boundary value problems of heat conduction. The analytical and numerical method of solutions of unsteady heat conduction problems are presented in many papers, see for example Refs. [5–10]. In this paper a mixed in time nonstationary problem of heat conduction for a homogeneous nondeformable half-space is considered. The half-space is assumed to be initially at zero temperature, the boundary plane is kept at a given temperature from time $t = 0$ to $t = t_0$, where t_0 is constant, and is insulated for $t > t_0$.

The mixed boundary value problems are well-known in literature in cases when a part of boundary is kept at a given temperature and heat flux is described on the remained part of boundary (see for instance: crack problems, contact problems).

2. Formulation and solution of the problem

Consider a nonstationary heat conduction problem for a rigid homogeneous half-space. Referring to the Cartesian coordinate system (x, y, z) with the plane $x = 0$ being the boundary surface of the half-space $x > 0$, denote at the point

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$\boldsymbol{x}=(x, y, z)$ the temperature (strictly the deviation of the temperature from a reference state) by T. Let the distribution of temperature dependent only on time t is assumed on the boundary plane for $0 \le t \le t_0$, and for $t > t_0$ the body is thermally insulated. Moreover, the zero initial temperature is assumed.

The considered problem is one-dimensional and is described by

(a) the equation of heat conduction [11]:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{k}\frac{\partial T(x,t)}{\partial t}, \quad x > 0,\ t > 0 \tag{2.1}$$

where k is the thermal diffusivity,

(b) the mixed-time boundary conduction

$$T(0,t) = T_0 f\left(\frac{t}{t_0}\right) \quad \text{for } 0 \le t \le t_0$$

$$\frac{\partial T(0,t)}{\partial x} = 0 \quad \text{for } t > t_0 \tag{2.2}$$

where $f(\cdot)$ is given function satisfying the following condition

$$f(0) = 0 \tag{2.3}$$

and T_0 is a given constant temperature.

(c) the initial condition

$$T(x,0) = 0 \quad \text{for } x \ge 0 \tag{2.4}$$

(d) the regularity condition in infinity

$$\lim_{x\to\infty} T(x,t) = \lim_{x\to\infty} \frac{\partial T(x,t)}{\partial x} = 0, \quad \mathrm{t} \ge 0. \tag{2.5}$$

Denoting by

$$\tau = \frac{t}{t_0} \tag{2.6}$$

the Eq. (2.1) with the boundary condition (2.2) take the form

$$\frac{\partial^2 T(x,\tau)}{\partial x^2} = \frac{1}{kt_0}\frac{\partial T(x,\tau)}{\partial \tau}, \ \text{for } x > 0, \tau > 0, \tag{2.7}$$

and

$$T(0,\tau) = T_0 f(\tau) \quad \text{for } 0 \le \tau \le 1$$

$$\frac{\partial T(0,\tau)}{\partial x} = 0 \quad \text{for } \tau > 1 \tag{2.8}$$

Using the cosine Fourier transform with the respect of x denoted by

$$\tilde{T}(\alpha,\tau) = \sqrt{\frac{2}{\pi}}\int_0^\infty T(x,\tau)\cos(x\alpha)\mathrm{d}x \tag{2.9}$$

we obtain from Eqs. (2.7), (2.8) and (2.5) the following linear ordinary differential equations

$$G(\tau) - \alpha^2 \tilde{T}(\alpha,\tau) = \frac{1}{kt_0}\frac{\mathrm{d}\tilde{T}(\alpha,\tau)}{\mathrm{d}\tau}, \quad \text{for } 0 \le \tau \le 1,$$