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Impacts of declining discount rates on optimal harvest age and land expectation values

Richard J. Brazee

Department of Natural Resources and Environmental Sciences, University of Illinois at Urbana-Champaign, W-503 Turner Hall, 1102 S. Goodwin, Urbana, IL 61801 USA

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ABSTRACT

Several national governments now require the discount rate for public projects with long planning horizons to decrease over the life of a project. Theoretical results that characterize the impacts of a declining discount rate on optimal harvest age and land expectation values in the Faustmann Model are presented. New results include: derivation of conditions for a sequences of optimal harvest ages and land expectation values, comparative statics results, conditions that must be satisfied for multiple optima, and proof that harvesting is never optimal immediately before a discrete decrease in the discount rate.

1 Introduction

Recently national governments including Denmark, France and the United Kingdom have adopted national policies that require the discount rate for public projects with long planning horizons to decrease over the life of each project.[1] Given the length of optimal rotations in temperate forests, optimal harvest ages for public forest projects will often need to be evaluated using two or more successively smaller discount rates. If forest land values are defined as the net present value (NPV) of as infinite stream of benefits as in Faustmann models, then land values for public projects will also need to be evaluated using two or more successively smaller discount rates for projects.

Previous studies simulate the impacts of a declining discount rate on optimal harvest age and land values (Hepburn and Koundouri 2007; Price, 2011, 2014, 2017).[2] The results of these studies differ from typical Faustmann results. In typical Faustmann models harvest age is constant across all rotations. With a declining discount rate optimal harvest age first increases as the discount rate declines, and then becomes constant after the discount rate becomes constant. Rotations of the same length generate different land values depending on when the rotation starts. The studies also note the possibility of multiple optimal harvest ages for a given rotation, which also contrasts with typical Faustmann models in which there is only one optimal harvest age for each rotation.

Although previous results are consistent with the Generalized Faustmann Model (Chang 1998) an important limitation of previous studies is the lack of a mathematically-based theoretic foundation for the simulation results. In these studies the existence of a sequence of optimal harvest ages and land expectation values is assumed rather than proved or described. Similarly, although simulations have

E-mail address: brazee@illinois.edu

[1] It matters not whether forest economists or other economists think that declining discount rates are a good idea, what is important, is that Denmark, France and the UK have adopted declining discount rates for evaluating public projects with long time horizons. In addition a Norwegian Expert Committee has recommended that Norway consider adopting a declining discount rate for public projects. The motivation for adopting a declining discount rate appears to vary by country. The motivation for the UK, the Norwegian Expert Committee and implicitly for Denmark appear are based on the decrease in uncertainty over time (HM Treasury, 2011; NOU, 2012; Finansministeriet, 2013; Price, unpublished). The motivation in France appears to be a predicted reduction in per capita income (Lebègue et al., 2005).

[2] A declining discount rate has been analyzed more extensively in the non-forestry economics literature. An incomplete list of previous research includes Arrow et al. (2013, 2014), Bayer (2003), Chichilnisky (1997), Cropper et al. (2014), Frederick et al. (2002); Gollier (2002), Gollier et al. (2008), Henderson and Bateman (1995), Kula (1981, 1984), Li and Löfgren (2000), Newell and Pizer (2003), Price (2004, 2005, 2008, 2011, 2014), Price and Nair (1985), Strotz (1956); Sumaila and Walters (2005), Weitzman (1998).

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Table 1
UK and France declining discount rate schedules.

Years	0–30	31–75	76–125	126–200	201–300	>300
France Discount Rate	8.0%	4.0%	4.0%	4.0%	4.0%	4.0%
U.K. Discount Rate	3.5%	3.0%	2.5%	2.0%	1.5%	1.0%

Table 2
Notation for a declining discount rate for the first rotation.

Name	Period 1	Period 2	∵	Period k_1
Length	$[t_0, t_1]$	$[t_1, t_2]$	∵	$\left[t_{k_i-1}, t_{k_i}\right]$
Period Discount Rate	r_1	r_2	∵	r_{k_1}
Cumulative Discount Rate	$r_1 a_{11}$	$r_1(t_1 - t_0) + r_2(a_{12} - t_1)$	∵	$\sum_{j=1}^{k_1-1} r_j(t_j - t_{j-1}) + r_{k_1}(a_{1k_1} - t_{k_1-1})$
Discount Factor	$e^{-r_1 a_{11}}$	$e^{-r_1(t_1-t_0)-r_2(a_{12}-t_1)}$	∵	$e^{-\sum_{j=1}^{k_1-1} r_j(t_j-t_{j-1})+r_{k_1}(a_{1k_1}-t_{k_1-1})}$

where
a_{1k_1} is the harvest age of the first rotation, when harvest is in period k_1,
k_1 is the period in which the first rotation is harvested,
r_j is the discount rate in period j,
r_{k_1} is the discount rate in the period in which the first rotation is harvested,
t_{j-1} is the start of period j, and
t_j is the end of period j.

reported multiple optimal harvest ages for a specific set of parameters, the conditions for when multiple optimal harvest ages for a specific rotation exist have not been demonstrated.

The first goal of this manuscript is to provide a mathematically-based foundation for previously reported simulation results. To meet this goal the Generalized Faustmann Model is extended, the conditions for a sequence of optimal harvest ages and land expectation values are demonstrated, comparative statics results for stumpage price, regeneration costs and each discrete declining discount rate are derived, and the general conditions which must hold for multiple optimal harvest ages to arise for a single rotation are described. The second goal is to demonstrate when the discount rate is declining in discrete steps as mandated by national laws it will never be optimal to harvest a rotation immediately before a decline in the discount rate.

2 Background and notation

The UK and France declining discount rate schedules are (HM Treasury 2011; Lebègue et al., 2005) (Table 1).

The schedule in Denmark is 4.0% for 0-35 years, 3.0% for 35-70 years and 2% after year 70. (Finansministeriet, 2013).

The inclusion of a declining discount rate greatly complicates the notation required for effective analysis. Table 2 presents notation that characterizes a declining discount rate for the first rotation of a Generalized Faustmann Model.

The first rotation is harvested during period k_1. Between t_{j-1} and t_j the discount rate is r_j which is the rate at which projects are discounted during period j. r_j may be thought as a marginal discount rate. The cumulative discount rate reflects the rate at which benefits realized at time k_1 are discounted to time 0. The cumulative discount rate for the first rotation is composed of two terms. The sum in the first term is the cumulative discount rate for the periods between time 0 when the first rotation is regenerated and the start of period k_1, when the first rotation is harvested. The second term is active for the portion of period k_i before harvest. The discount factor is the exponential of the cumulative discount rate.

Similar to much of the previous research on optimal harvesting with a declining discount rate, we adopt standard assumptions from typical Faustmann Models including the objective of maximizing the NPV of bare land from timber production with respect to a set of harvest ages, constant regeneration costs, a constant stumpage price, and a concave volume function under even-aged management. Although some of these assumptions are not required in a Generalized Faustmann Model, retaining these assumptions simplifies the analysis.[3]

With a declining discount rate both optimal harvest ages and land values vary over the planning horizon depending on when a rotation starts. In preparation for deriving conditions for a set of optimal harvest ages using an extended Generalized Faustmann Model, a useful way to write the land value at the start of the first rotation when harvest is in period k_i is:

$$L(0, k_1) \equiv \begin{matrix} maximum\ NPV \\ w.r.t.\ a_{1k_1} \end{matrix} = e^{-\sum_{j=1}^{k_1-1} r_j(t_j-t_{j-1})+r_{k_1}(a_{1k_1}-t_{k_1-1})}[pV(a_{1k_1}) + LEV(a_{1k_1})] - C \tag{1a}$$

where, C is the regeneration costs of a rotation,

$L(0,k_1)$ is the bare land value at the start of the first rotation if the rotation is harvested in the period k_1 in a Generalized Faustmann Model,

[3] There are several hundred papers that study or apply the Faustmann Model. Foundational papers include Faustmann (1849), Pressler (1860), Ohlin (1921), Samuelson (1976). For a review see Amacher et al. (2009). Chang (1998) is the foundational paper for the Generalized Faustmann Model.

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