Journal of Forest Economics 30 (2018) 32–37

Contents lists available at ScienceDirect

Journal of Forest Economics

journal homepage: www.elsevier.com/locate/jfe

On Pressler's indicator rate formula under the generalized Reed model

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ARTICLE INFO

Article history:
Received 10 August 2017
Accepted 22 December 2017

Keywords:
Pressler
Generalized Reed model
Natural disturbances
Risk
Timber salvage
Harvest age

ABSTRACT

We extend the Pressler's indicator rate formula under the generalized Reed model to account for the impacts of current and future stochastic disturbance risk on the current harvesting decision. We prove that that the mathematical framework of the Pressler's indicator rate holds under the generalized Reed model. We apply it to the management of longleaf pine to determine the optimal harvest age under the risk of wildfires. We determine that the Pressler's indicator rate formula provides a useful framework to determine the minimum timber salvage increment required to decide when to harvest longleaf pine under the risk of wildfire.

Introduction

The indicator per cent formula developed by Max Pressler (Pressler, 1860) is considered a useful indicator of the economic maturity of a forest stand (Gong and Löfgren, 2010). In his seminal paper, Pressler recognizes that the variation in the timber value of a forest stand is caused by the quantity increment, quality increment, and price increment. The sum of the rates of these three increments adjusted for the timber value and land value represents Pressler's indicator rate. As proven by Johansson and Löfgren (1985), Pressler's indicator rate represents the first order condition for the maximization of the Faustmann-based land expectation value to determine optimal harvest age.

Catastrophic natural events (e.g., wildfires, storms, pest outbreaks) continually shape the forest landscape. They also have a direct economic impact on forest landowner decisions and value of a forest stand. One of the most notable studies in this area was explored by Reed (1984), who adapted the basic Faustmann model to incorporate the impacts of stochastic natural disturbances using a Poisson jump process on the optimal harvest age of a stand. Unlike the traditional Reed approach, Susaeta et al. (2016) developed a model in which the risk of natural hazards, timber salvage of the damaged timber crop, and economic and biological parameters of the model (stumpage prices, forest growth, discount rate and regeneration costs) may vary after successive forest rotations.[1] Thus, this more flexible model allows the optimal harvest age to change over successive timber crops. Although Pressler's mathematical framework also holds under the generalized Faustmann formula including, respectively, timber and carbon benefits, from a forest management perspective it is relevant to determine the practical usefulness of this indicator rate formula under a generalized version of the Reed model (GRM henceforth).

This paper explores whether Pressler's indicator rate formula is applicable under such a dynamic and stochastic environment to determine optimal harvest age, and how the impacts of current or future decisions in the presence of increased natural disturbances can affect the landowner's harvesting decision. We also consider that the risk of rare natural disturbances can be described using the Poisson distribution since it only requires historical data on the frequency of natural disturbances (Amacher et al., 2009) and has a clear physical and probabilistic meaning, allowing an easy interpretation of the hazard event (Mandallaz and Ye, 1997).[2] The remainder of this paper is as follows: First, the underlying features of the GRM are outlined. Next, the Pressler's indicator rate formula is developed under the risk of natural disturbances. Then, the indicator is applied to a representative southern species, longleaf pine

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[1] For example, the risk of natural disturbances is expected to increase over time due to changing climatic conditions (Wear and Greis, 2012), and poor future economic conditions can negatively affect timber salvage operations (Prestemon and Holmes, 2008).

[2] Although the Poisson stochastic process has been widely employed to model rare natural hazards such as winds, wildfires, droughts, and insect outbreaks (Amacher et al., 2005; Loisel, 2014; Taylor et al., 2013), other probability distributions such as the Weibull distribution (Staupendahl and Mohring, 2011), binomial distribution (Diaz-Balteiro et al., 2014) and Gumbel distribution (Blennow and Oloffson, 2008) have been suggested to model natural disturbances. The use of non-parametric process has also been employed to model the arrival of hazard events (Deegen and Matolepszy, 2015).

https://doi.org/10.1016/j.jfe.2017.12.002
1104-6899/

(*Pinus palustris* Mill.), and the results are discussed. Conclusions and recommendations for further research are offered in the last section.

The generalized Reed model

We present here the key relationships of the GRM. Further details of the GRM can be found in Susaeta et al. (2016). We assume that a forest stand may be damaged by a natural disturbance before reaching the optimal harvest age. After the catastrophic event, the forest landowner can salvage a proportion of the stand g and replant a new forest stand. If the forest stand has reached the optimal harvest without being affected by a natural disturbance, the landowner harvests the stand and replants to begin a new rotation. The probability of arrival (or arrival rate λ) of a natural disturbance follows a non-homogenous Poisson distribution process, i.e. it dependent on the age of the stand, thus $\lambda = \lambda(t)$.

It is assumed that the time between successive catastrophic events is a random variable x (the age of the stand when a disturbance occurs), exponentially distributed with cumulative density function $1 - e^{-m(x)}$, where $m(x)$ represents the cumulative sum of the probability of arrival of a natural disturbance over time: $m(x) = \int_0^x \lambda(q)dq$ and is increasing in x, thus $\frac{dm}{dx} = \lambda(x)$. We have considered that the probability of arrival increases over time, i.e., $\lambda'(x) > 0$. The probability density function of x before reaching the optimal rotation age $T(0 < x < T)$ is given by $\lambda(x)e^{-mx}$. The probability that a disturbance event may affect a forest stand before reaching the economically optimal rotation age T is $Pr(x < T) = 1 - e^{-mT}$ and the probability of the stand reaching the optimal rotation without being affected by a disturbance event is $Pr(x = T) = e^{-mT}$. We also assume that the forest stands are independent from each other with respect to the risk the risks of catastrophic events; i.e., our model applies only in situations where the risk of natural disturbances does not depend on how the neighboring stands are managed.[3]

We also consider that, regardless of the arrival of a natural disturbance, for each timber crop, a landowner will have to replant a new timber crop, with different levels of stumpage prices and forest growth, risk of natural disturbances, replanting costs, discount rates and salvageable portions. As such, the optimal harvest age will fluctuate from timber crop to timber crop. The parameters of the GRM model are defined in Table 1. The expected net present value of harvesting the stand at the optimal harvest age or salvaging a portion of the stand for the first timber crop Y_1 is

$$Y_1 = -C_0 + [V_1(t_1) - C_1 + e^{(r_1t_1+m_1(t_1))}\int_0^{t_1} \emptyset_1 dx_1]e^{-(r_1t_1+m_1(t_1))} \quad (1)$$

In Eq. (1), the landowner incurs the regeneration costs C_0 at the beginning of the first timber crop. The landowner will obtain the following economic revenues for the first timber crop: a) timber benefits $V_1(t_1)$, i.e., the product between the stumpage price $P_1(t_1)$ and the volume of the stand $Q_1(t_1)$ at the time of harvesting if the stand has not been affected by a natural disturbance; and 2) net timber revenues due to salvage $\int_0^{t_1} \emptyset_1 dx_1$, i.e., the cumulative sum of the present value of the marginal returns due to timber salvage $\emptyset_1 = \lambda_1(x_1)[\bar{g}_1(x_1)V_1(x_1) - C_1]e^{-(r_1x_1+m_1(x_1))}$ if a natural disturbance arrives before the optimal harvest age. After harvesting or salvaging a portion of the stand, the landowner will incur replanting costs C_1 to start the second timber crop. All future net economic revenues are discounted accordingly using the term $e^{-(r_1t_1+m_1(t_1))}$ to obtain the expected net present value for the timber crop.[4] The same treatment applies for successive timber crops. Under the GRM, the land expectation value at the beginning of the ith timber crop is defined as LEV_i. The land expectation value at the beginning of the first timber crop LEV_1 is as follows:

$$\begin{aligned} LEV_1 &= -C_0 + [V_1(t_1) - C_1 + e^{(r_1t_1+m_1(t_1))}\int_0^{t_1} \emptyset_1 dx_1]e^{-(r_1t_1+m_1(t_1))} + [V_2(t_2) - C_2 + e^{(r_2t_2+m_2(t_2))}\int_0^{t_2} \emptyset_2 dx_2]e^{-(r_1t_1+m_1(t_1))}e^{-(r_2t_2+m_2(t_2))} + \ldots \\ &= -C_0 + \sum_{i=1}^{\infty} [V_i(t_i) - C_i + e^{(r_it_i+m_i(t_i))}\int_0^{t_i} \emptyset_i dx_i]e^{-\sum_{j=1}^{i}(r_jt_j+m_j(t_j))} \end{aligned} \quad (2)$$

Eq. (2) represents the sum of the expected economic returns – due to either harvesting or salvaging timber – under the risk of a natural disturbance associated with infinite number of successive timber crops. The expected value of each timber crop is discounted to the beginning of the first timber crop using the term $e^{-\sum_{j=1}^{i}(r_jt_j+m_j(t_j))}$, where i and j are indexes to represent timber crops. Eq. (2) can be re-written as follows:

$$\begin{aligned} LEV_1 &= -C_0 + [V_1(t_1) + e^{(r_1t_1+m_1(t_1))}\int_0^{t_1} \emptyset_1 dx_1]e^{-(r_1t_1+m_1(t_1))} + e^{-(r_1t_1+m_1(t_1)}\left\{-C_1 + \sum_{i=2}^{\infty} [V_i(t_i) - C_i + e^{(r_it_i+m_i(t_i))}\int_0^{t_i} \emptyset_i dx_i]e^{-\sum_{j=2}^{i}(r_jt_j+m_j(t_j))}\right\} \\ &= C_0 + [V_1(t_1) + e^{(r_1t_1+m_1(t_1))}\int_0^{t_1} \emptyset_1 dx_1]e^{-(r_1t_1+m_1(t_1))} + e^{-(r_1t_1+m_1(t_1))}LEV_2 \end{aligned} \quad (3)$$

Eq. (3) suggests the linkage of land values between successive timber crops: LEV_1 depends on LEV_2, LEV_2 depends on $LEV_3$and so on. The full derivation of the conditions for reaching the optimal harvest age can be found in Susaeta et al. (2016). Here, we present the first order condition for the optimal harvest age t_i:

$$\begin{aligned} \frac{\partial LEV_1}{\partial t_i} &= \frac{\partial V_i(t_i)}{\partial t_i} + \lambda_i(t_i)[\bar{g}_i(t_i)V_i(t_i) - C_i] - [r_i + \lambda_i(t_i)]V_i(t_i) + [r_i + \lambda_i(t_i)]LEV_{i+1} = 0 \\ \frac{\partial V_i(t_i)}{\partial t_i} &+ \lambda_i(t_i)[\bar{g}_i(t_i)V_i(t_i) - C_i] = [r_i + \lambda_i(t_i)]V_i(t_i) + [r_i + \lambda_i(t_i)]LEV_{i+1} \end{aligned} \quad (4)$$

[3] Other studies have suggested that neighboring stand structures can affect the probability of arrival of natural disturbances (Blennow and Oloffson, 2008; Lohmander, 1987; Ziegler et al., 2017). Lohmander (1987) determined that the windthrow probability is strongly dependent on the tree height and stand density in the neighbor stand. Deegen and Matolepszy (2015) found that, under storm risk, developing protection management practices in neighboring stands increases the profitability of the stand, and extends the optimal harvest age in the presence of low timber prices.

[4] The discount factor $e^{-(r_1t_1+m_1(t_1))}$ also acts as a common denominator for the terms between [] in Eq. (1). However, the timber revenue due to salvage (the integral term) is already discounted due to the definition of ϕ_1. Therefore, we need to use the compounding factor $e^{(r_1t_1+m_1(t_1))}$ for the integral term to maintain the present value of the timber salvage revenues. This is simply illustrated by the following: $Y_1 = -C_0 + [V_1(t_1) - C_1]e^{-(r_1t_1+m_1(t_1))} + \int_0^{t_1} \emptyset_1 dx_1 = -C_0 + [V_1(t_1) - C_1 + e^{(r_1t_1+m_1(t_1))}\int_0^{t_1} \emptyset_1 dx_1]e^{-(r_1t_1+m_1(t_1))}$.