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Optimal rotation with negative discount rates: completing the picture

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ABSTRACT

While the rotation of maximum forest rent (no discounting of revenues and costs) is sometimes discussed conceptually and calculated practically, forest economists have almost invariably focused on optimal rotation with a positive discount rate. Some circumstances, especially related to increasing marginal utility with decreasing availability per head of products, would justify use of a negative discount rate. For a logically complete suite of conditions, a means of identifying optimal rotation with such a rate is needed. Applying the usual formulation for land expectation value then produces a meaningless result. Two approaches seem to give a solution. One uses the first-order conditions for an optimum. Another uses inversion of time perspective to produce sensible-seeming results. However, for cases where the world, or the value of forest production, ends suddenly at an unpredictable time, the optimal rotation is, surprisingly, that of maximum forest rent. Where the cause of negative discount rates is of limited duration before stability or growing affluence is re-established, this "provisional optimum" can be modified responsively.

Introduction

Almost invariably, when forest economists discuss optimal rotation, they do so in the context of a positive discount rate. The fame of Martin Faustmann (1849) lies in his derivation of a formula for this context, giving the land expectation value (LEV) of a perpetual series of forest rotations. Using this formula, the "correct" – i.e. financially most profitable – rotation may be identified. Faustmann himself, writing as "F" (1849), did so in print. The formula has more recently been "discovered" by mainstream economists (Samuelson, 1976).

Reduced to essentials, the Faustmann formula for LEV is

$$\mathrm{LEV} = \frac{V_T\,e^{-\rho T} - C_0}{1 - e^{-\rho T}} \tag{1}$$

where C_0 is the cost at the beginning of the rotation,
V_T is the revenue at the end, T, of the rotation
and ρ is the discount rate.

To simplify presentation in this paper, all intermediate cash flows have been omitted, but their inclusion would not change the argument.

An algebraic result for optimal rotation is given by differentiating the formula and setting the result equal to 0. This approach has often been discussed, using the term "first-order condition" (Johansson and Löfgren, 1985). However, unless V_T is a rather simple function of time, the value T^* giving the maximum LEV can most readily be found numerically.

Some forest economists have also explored the case for not discounting at all, and the implications for optimal rotation length (Markus, 1967). The result is derived from the forest rent (FR) (i.e. net annual revenue), given as

$$\mathrm{FR} = \frac{V_T - C_0}{T}. \tag{2}$$

Again, algebraic or numerical results for the rotation of maximum forest rent can be determined. It has been pointed out that, under certain constrained circumstances discussed later, this may be the most valuable rotation, even with positive discounting (Tait, 1987).

Further, I have previously discussed the optimal sequence of rotations with declining discount rate (Price, 2011a). Such a decline could in theory include decline to – or below – zero.

This paper focuses on defining optimal rotation with a negative discount rate, taking a different perspective from that of Johansson and Löfgren (1985, chapter 5), who also addressed the problem. It considers briefly the cases for and against using a negative discount rate, but is largely occupied with the problem of what the optimal rotation would be with such a rate. The existence of a solution is, indeed, an answer to one of the objections against using negative discount rates for determining optimal rotations – the (incorrect) belief that "it can't be done".

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What a negative discount rate is, and is not

A positive discount rate means that some defined entity or process occurring in future is considered *less* valuable than the same thing occurring presently: a negative discount rate means that it is considered *more* valuable. It does *not* mean that *less* is preferred to *more*: but that, in the defined circumstances, *later* is preferred to *earlier*. To be defensible, this position (like its converse) must be based on some changes taking place through time that would systematically alter value, rather than on psychological time preference or deemed precedence of one generation over another: I would no more recommend such an absurd thing as giving future consumption a premium *merely* because of its futurity, than I would countenance ascribing to it a discount *merely* because of its futurity.

Several other reasons have been proposed, though not universally accepted, for discounting at a positive rate: for example, early consumption brings forwards the availability of choice, preempts change of taste, and grants early fulfilment of desires. These are invalid as general arguments for discounting, and may indeed sometimes be adduced to justify *greater* value to later consumption: more informed choice, maturation of taste, anticipation of gratification – it was no less an authority than Marshall (1891) who said "When calculating the rate at which a future benefit is discounted, we must be careful to make allowance for the pleasures of expectation." Uncertainty of natural systems may also justify a negative discount rate (Drepper and Månsson, 1993). It is, moreover, of interest that arguments for *lower* discount rates have been derived from uncertainty (Weitzman, 1998; Newell and Pizer, 2004), which was traditionally viewed as a reason for *higher* discount rates. These arguments for ascribing enhanced future values, it is worth noting, tend to be associated with a more thoughtful, less instinctive, process of valuation, one in which "the planner" gains ascendancy over "the doer" (Thaler and Shefrin, 1981).

The most ethically defensible case for discounting arises from the presumed diminishing marginal utility of consumption, as technological advance and the accumulation of capital have allowed, mostly, increasing consumption per head (Tullock, 1964; Schelling, 1999; Price, 2003). But the opposite case, for *increasing* marginal utility, might arise from countervailing forces of growing population, bearing on resource limitations and finite pollution absorption capacity (Price, 1973). And even if, for humankind in general, consumption per head has historically increased, for some groups and during some periods it has not done, and under some future scenarios it would not do.

The uncertainties surrounding climate change, as well as distributional issues concerning who bears the costs, have also brought suggestions for negative discount rates (Dasgupta et al., 1999; Fleurbaey and Zuber, 2013). It is of relevance that climate change mitigation is widely seen as an important future role for forests, with major benefits accruing to end-of-rotation fossil fuel displacement.

From a different perspective, the diminishing marginal utility of increasing cash income, and the limited substitutability of cash for such effects as personal injury, have meant that zero and negative discount rates have been advocated for compensation payments (respectively, by Broome (1994), Price (2000)).

While accepting all this, some have argued that people would not invest at negative (real) interest rates. However, the facts are that base interest rates have in real terms been negative during recent times. On 20th March 2017, the UK government's Ministry of Justice (2017) revised its discount rate for the settlement of injury claims down from 2.5% to −0.7%, on the grounds of negative yield from risk-free investment, and perhaps also the increasing cost of care. Despite negative rates, people have continued to invest, perhaps on the grounds of potentially increasing marginal utility or of uncertainty concerning the future (Price, 1993, chapter 11). In any case, the discount rate is not the same as the interest rate, conceptually or numerically.

What is logically possible and empirically observable should be susceptible to technical analysis. We should not allow ourselves to be obstructed from investigating a situation for lack of analytical tools. And, as it turns out, the results of the analysis are surprising.

When the ideas presented below were first offered for discussion, 34 years ago, the world was still by and large convinced that marginal utility was logically and historically fated to continue to diminish; that net rates of return on investment (even after including all the social and environmental externalities) were bound to remain positive; and that natural resource limitations would always be overwhelmed by advancing technology. Although this remains the position of most mainstream economists, the confidence that all manner of things shall not only *be well*, but shall always *become better*, *throughout all periods*, has been jolted as much by financial turmoil as by climate change and resource shortages. The prospects for cheap, safe and abundant nuclear fusion energy still look much as they did 62 years ago, when von Neumann (1955) predicted that, within a few decades, energy would be so cheap as to be not worth charging for.

Such lines of reasoning have given due cause for negative discount rates to be mentioned more often in the literature in recent times, with some examples noted above. What might once have seemed wild sooth-saying now emerges from factual observation and neoclassical discourse.

One further argument offered against calculating optimal rotations with negative discount rates is that "it can't be done". As has been demonstrated already, and is about to be demonstrated further, this is simply not true, at least, not for *moderately* negative discount rates.

Can it be done?

Nothing changes in the *mechanics* of discounting, just because discount rates, or times of cash flow, or both, take a negative value. Thus the discounted value of V_t received t years into the *future* at a continuous discount rate of ρ is

$$V_t e^{-\rho t} \tag{3}$$

It is only necessary to insert the appropriate negative sign for discount rate: for example if V_t is €1000, ρ is −3% and t is 50, the discounted value is

$$€1000 \times e^{-(-0.03)\times 50} = €4482 \tag{4a}$$

The intended and expected result is achieved: the future value is greater than the value of the same entity or process received or expended at present. Similarly, the present equivalent of €1000 occurring 50 years *in the past* with ρ = +3% is

$$€1000 \times e^{-0.03\times(-50)} = €4482 \tag{4b}$$

and the present equivalent of €1000 occurring 50 years in the past with ρ = −3% is

$$€1000 \times e^{-(-0.03)\times(-50)} = €223 \tag{4c}$$

We can reduce ambiguity in all the following transformations of value through time by using Eq. (3) both for conventional discounting, and for compounding forwards to a reference date, in which case the time of cash flow, t, takes a negative sign.

Despite this absolute consistency with sense and expectation, difficulties begin when negative discounting is combined with an infinite time horizon, as used in classical forest economics. The Faustmann formula for the NPV of a perpetual series of rotations, often termed LEV (land expectation value), is derived from the sum-