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Double diffusion from a vertical truncated cone in a non-Newtonian fluid saturated porous medium with variable heat and mass fluxes ☆

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ABSTRACT

This paper studies the double diffusion flow over a vertical truncated cone with variable heat and mass fluxes in a porous medium saturated with non-Newtonian power-law fluids. A coordinate transformation is used to obtain the nonsimilar governing equations, and the transformed boundary layer equations are then solved by the cubic spline collocation method. Results for local surface temperature and concentration are presented as functions of power-law indexes, exponents for variable heat and mass fluxes, buoyancy ratios, and Lewis numbers. The local surface temperature and concentration of the truncated cone decrease as the exponents for variable heat and mass fluxes are increased. Moreover, a decrease in the power-law index of fluids tends to decrease the local surface temperature and concentration of the truncated cone.

1. Introduction

Non-Newtonian power-law fluids in a porous matrix have nonlinear characteristics that are quite different from those of Newtonian fluids in porous media. The double diffusion flow of non-Newtonian fluids in porous media is of much importance because of numerous engineering applications, such as oil recovery and food processing. Chen and Chen [1] obtained similarity solutions for free convection of a non-Newtonian fluid over spheres and cylinders in porous media. Nakayama and Koyama [2] studied the natural convection of a non-Newtonian fluid over a non-isothermal body of arbitrary shape embedded in a porous medium. Rastogi and Poulikakos [3] examined the problem of double diffusion from a plate in a porous medium saturated with a non-Newtonian power-law fluid. Getachew et al. [4] performed a numerical and theoretical study of double-diffusive natural convection in a rectangular porous cavity saturated by a non-Newtonian power-law fluid. Benhadji and Vasseur [5] studied the double-diffusive convection in a shallow porous cavity filled with a non-Newtonian fluid. Kim and Hyun [6] studied the natural convection heat transfer of a power-law fluid in an enclosure filled with heat-generating porous media. Hadim [7] examined the non-Darcy natural convection of a non-Newtonian fluid in a porous cavity.

There are studies about the natural or mixed convection from cones for Newtonian fluids and micropolar fluids. Na and Chiou [8] studied the laminar natural convection of Newtonian fluids over a frustum of a cone. Wang et al. [9] examined the mixed convection from a rotating cone with variable surface temperature. Yih [10] examined the effect of radiation on natural convection of Newtonian fluids about a truncated cone. Pop and Na [11] presented the natural convection of Newtonian fluids over a vertical wavy frustum of a cone. Chamkha [12] also examined the coupled heat and mass transfer by natural convection of Newtonian fluids about a truncated cone in the presence of magnetic field and radiation effects. Postelnicu [13] studied the free convection about a vertical frustum of a cone in a micropolar fluid with constant wall temperature. Cheng [14] studied the heat transfer by natural convection of a micropolar fluid from a vertical truncated cone with power-law variation in temperature.

There are a few studies about the natural convection from cones in porous media saturated with Newtonian or non-Newtonian fluids. Cheng et al. [15] studied the problem of natural convection of a Darcian fluid about a cone. Yih [16] examined the coupled heat and mass transfer by free convection over truncated cone in porous media for variable wall temperature and concentration as well as for variable heat flux and variable mass flux. Cheng [17] studied the problem of heat and mass transfer by natural convection from truncated cones in porous media saturated with a non-Newtonian fluid with variable wall temperature and concentration.

This article tends to extend the work of Yih [16] and Cheng [17] to the problem of double diffusion flow over vertical truncated cone in porous media saturated with non-Newtonian power-law fluids with variable heat and mass fluxes. Thus this work aims to study the double diffusion near a vertical truncated cone in a power-law fluid embedded in a porous medium with variable heat and mass fluxes. A coordinate transformation is used to derive nonsimilar governing equations, and the obtained boundary layer equations are then solved by the cubic spline collocation method [18,19]. The effects of the Lewis number, the buoyancy ratio, the power-law index, and the exponent for variable heat and mass fluxes on the local surface temperature and

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Nomenclature

A	half angle of the frustum cone
a, b	constants
C	concentration
D	mass diffusivity
f	dimensionless stream function
g	gravitational acceleration
h	local heat transfer coefficient
h_m	local mass transfer coeficient
K	modified permeability of porous medium
Le	Lewis number
m_w	local mass flux
m	exponent for variable heat and mass fluxes
n	power-law index
N	buoyancy ratio
Nu	local Nusselt number
q_w	local heat flux
r	local radius of the truncated cone
Ra	modified Rayleigh number
Sh	local Sherwod number
T	temperature
u, v	Darcy velocity components
x, y	Cartesian coordiates
$\bar{x}$	distance measured from the leading edge of the truncated cone
x_0	distance between the leading edge of the truncated cone and the origin

Greek symbols

α	thermal diffusivity
β_c	coefficient of concentration expansion
β_T	coefficient of thermal expansion
η, ξ	dimensionless coordinates
θ	dimensionless temperature
μ_{eff}	effective viscosity
ρ	density
ϕ	dimensionless concentration
ψ	stream function

Subscripts

w	condition at wall
∞	condition at infinity

concentration from a vertical truncated cone in a porous medium saturated with power-law fluids with variable heat and mass fluxes are carefully examined.

2. Analysis

We consider the steady-state two-dimensional, laminar double diffusion boundary layer flow near a vertical truncated cone in a porous medium saturated with a non-Newtonian power-law fluid, as shown in Fig. 1. The origin of the coordinate system is placed at the vertex of the cone, where x is the coordinate along the surface of the cone measured from the origin and y is the coordinate normal to the surface. The surface of the truncated cone is held at a non-uniform heat flux $q_w(x)$ with the ambient fluid temperature being T_∞. The mass flux of a certain constituent in the solution that saturates the porous medium held at $m_w(x)$ on the fluid side of the surface of the truncated cone with the concentration sufficiently far from the surface of the cone being C_∞.

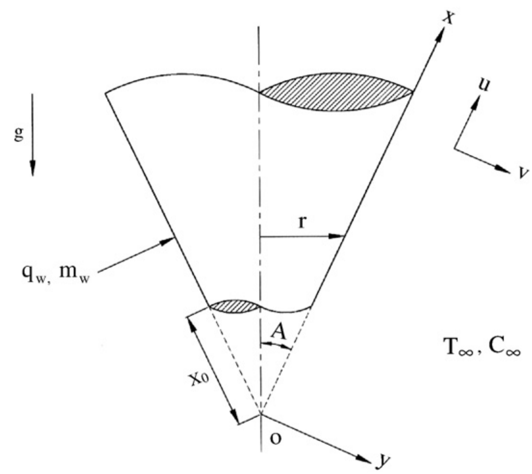

Fig. 1. Physical model and coordinates for a truncated cone.

The fluid properties are assumed to be constant except for density variations in the buoyancy force term. With introducing Boussinesq approximations, the equations governing the steady-state conservation of mass, momentum, energy, and constituent for laminar boundary layer flow through a porous medium saturated with a non-Newtonian power-law fluid near the surface of the truncated cone can be written in two-dimensional Cartesian coordinates (x, y) as [16,17]

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0 \tag{1}$$

$$\frac{\mu_{eff}}{K} u^n = \rho g \beta_T (T - T_\infty)\cos A + \rho g \beta_C (C - C_\infty)\cos A \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

The appropriate boundary conditions for the problem are:

$$v = 0, -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = q_w = a(x - x_0)^m, \tag{5}$$

$$-D\left(\frac{\partial C}{\partial y}\right)_{y=0} = m_w = b(x - x_0)^m \text{ on } y = 0, x_0 \le x \le \infty$$

$$u = 0,\ T = T_\infty,\ C = C_\infty \text{ as } y \to \infty \tag{6}$$

Here m is the exponent for variable heat and mass fluxes, and n is the power-law index of the fluid. a and b are constants. u and v are the Darcian velocity components in the x and y directions, respectively. x_0 is the distance of the leading edge of the truncated cone measured from the origin and r is the radius of the truncated cone. The modified streamwise coordinate $\bar{x}$ is defined as $\bar{x} = x - x_0$. T and C are the volume-averaged temperature and concentration, respectively. q_w and m_w are the local heat flux and local mass flux, respectively. α and D are the equivalent thermal and mass diffusivity of the saturated porous medium, respectively, β_T and β_C are the coefficients for thermal expansion and for concentration expansion of the saturated porous medium, respectively. μ_{eff} and K are the effective viscosity and the modified permeability of the porous medium, respectively. ρ and g are the density and the gravitational acceleration., respectively.