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Particle transport in turbulent flow using both Lagrangian and Eulerian formulations ☆

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ABSTRACT

Lagrangian and Eulerian models for particle transport by a turbulent fluid phase are presented. In both methods, particle distribution results from the action of applied forces (buoyancy, inertial, added mass and drag forces) and turbulent effects are shown. The carrier phase flow – which is solved by finite element method using a k–ε turbulence model – is assumed not to depend on the particles' motion. In the Lagrangian formulation the dynamic equation for the particles is solved. A discrete random walk model is used to account for the turbulent effects. In the Eulerian formulation, the particle concentration is calculated from a convection–diffusion equation using the terminal particles' velocity and turbulent diffusivity. Both models are compared to experimental measurements and analytical results; a good agreement is observed.

1. Introduction

Particle transport by a fluid carrier is a phenomenon of great interest since it is frequently found in many scientific fields and industrial processes. Particle dispersion and deposition plays a critical role in several scientific fields and industrial applications:

- inclusion transport in liquid steel [1,2],
- pharmaceutical particle transport and deposition in the human lung [3],
- particle separation in a hydrocyclone [4],
- pollutant transport and deposition in coastal waters [5],
- aerosol transport in flows [6],
- dust pollution in urban area [7], and
- dynamic behavior of dross in hot dip bath [8].

The numerical simulation of particle transport by a fluid carrier requires the modeling of the continuous phase (fluid), the discrete phase (particles) and the interaction between them. The continuous phase – whether liquid or gas – is modeled using a Eulerian formulation. The discrete phase – fluid, gaseous or solid – may be approached both from a Eulerian or from a Lagrangian point of view. This has given place to two distinctive strategies, the so called Eulerian–Eulerian and the Eulerian–Lagrangian methods. In the Eulerian–Eulerian approach [4,6,7,9–11], particle velocity and concentration fields are calculated for each point of the numerical domain. The Eulerian–Eulerian method can be employed both using a one-fluid formulation [2,4] and a two-fluid formulation [6,7,9–11]. In the latter, mass conservation and momentum conservation equations are used to calculate the particle concentration and velocity fields, so the phases are treated as two interpenetrating fluids which interact through their interfacial properties. In the one-fluid formulation, on the other hand, no momentum conservation equation is used. Particle velocity is usually determined by an algebraic equation for the particle–fluid slip velocity. In the Eulerian–Lagrangian formulations [5,12–15], each particle trajectory is simulated. The particle dynamic is generally governed by the Basset–Boussinesq–Oseen (BBO) equation and a random walk model is applied to account for the turbulence effect. In order to solve the BBO equation, the continuous phase properties must be calculated at particle position.

The interaction between phases is modeled according to the strength of the coupling between them. For dense particle concentration, particle action on the fluid and the interaction between particles must be accounted for (four way coupling). For intermediate concentrations, particle interaction can be neglected but particle influence on the continuous phase cannot (two-way coupling). For dilute concentrations, the fluid flow may be considered regardless of the particles flow (one-way coupling). According to Elghobashi [16]), a criterion to determine the type of interaction in terms of the particle volume fraction of particles (α^p) is as follows:

- $\alpha^p < 10^{-6}$, for one-way coupling [1–15].
- $10^{-6} \leq \alpha^p \leq 10^{-3}$, for two-way coupling [17,18].
- $\alpha^p > 10^{-3}$, for four-way coupling [19].

In the present work, we will present both a Eulerian–Eulerian – one-fluid – formulation and a Eulerian–Lagrangian formulation and we will use them to solve the same problem. Since both formulations may provide useful information about the particle transport process,

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Nomenclature

C_μ	Constant of k–ε model
C_1	Constant of k–ε model
C_2	Constant of k–ε model
C_D	Drag coefficient
c^p	Dimensionless concentration
C^p	Particle mass concentration
C_c^p	Centerline particle mass concentration
C_o^p	Initial particle mass concentration
D^p	Particle diameter
Dw	Particle–wall distance
$\mathbf{g}$	Gravity acceleration
h	Height
I_o	Modified Bessel function of first kind and order 0
J_v	Bessel function of first kind and order v
k	Turbulent kinetic energy
L	Mixing length
m^p	Particle mass
N^p	Number of particles
P	Time averaged pressure
R	Radius
$\mathfrak{Re}^p$	Particle Reynolds number
t	Time
T	Relaxation time
$\mathbf{v}^f$	Time average fluid velocity
$\mathbf{v}_0^f$	Uniform axial fluid velocity
$\mathbf{v}^p$	Particle velocity
$\mathbf{v}_0^p$	Particle velocity initial value
$\mathbf{v}_T^p$	Particle velocity terminal value
$\mathbf{w}$	Normal Gaussian random variable
$\mathbf{x}^p$	Particle position
z_m	Zeros of J_o

Greek symbols

α^p	Particle volume fraction
Δt	Time step
ε	Turbulent kinetic energy dissipation rate
μ^f	Fluid laminar viscosity
η	Dimensionless radial coordinate
ρ^f	Fluid density
ρ^p	Particle density
σ_k	Constant of k–ε model
σ_ε	Constant of k–ε model
σ	Turbulent Schmidt number
θ	Dimensionless time
ξ	Dimensionless position

our motivation is to show the consistency of both approaches by comparing their results to measurements of particle concentration in free turbulent axisymmetric jet. In the following section the turbulent flow model is presented. In Section 3, we describe the Lagrangian and Eulerian formulations. Section 4 contains a series of simple problems used for model verification. In Section 5, the methods are validated by comparison with experimental concentration measurements. Section 6 is devoted to conclusions.

2. Turbulent flow model

The general flow is assumed not to depend on particle dynamics; therefore it is uncoupled from the particle transport model (described in Section 3).

The turbulent flow model hypotheses are viscous incompressible flow, isothermal flow, constant fluid density, constant fluid laminar viscosity and a turbulence k–ε model. The following equations are solved:

$$\nabla \cdot \mathbf{v}^f = 0 \tag{1}$$

$$\rho^f \frac{\partial \mathbf{v}^f}{\partial t} + \rho^f \mathbf{v}^f . \nabla \mathbf{v}^f - \nabla \cdot \left[\left(\mu^f + \mu^t \right) \left(\nabla \mathbf{v}^f + \nabla \mathbf{v}^{f^T} \right) \right] + \nabla P = 0 \tag{2}$$

$$\rho^f \frac{\partial k}{\partial t} + \rho^f \mathbf{v}^f \cdot \nabla k - \nabla \times \left[\left(\mu^f + \frac{\mu^t}{\sigma_k} \right) \nabla k \right] - \mu^t \left(\nabla \mathbf{v}^f + \nabla \mathbf{v}^{f^T} \right) : \nabla \mathbf{v}^f + \rho^f \frac{C_\mu k^2}{\mu^t / \rho^f} = 0 \tag{3}$$

$$\mu^t = C_\mu \rho^f \sqrt{k} L \tag{4}$$

$$\rho^f \frac{\partial \varepsilon}{\partial t} + \rho^f \mathbf{v}^f \cdot \nabla \varepsilon - \nabla \times \left[\left(\mu^f + \frac{\mu^t}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] - \rho^f C_\mu C_1 k \left(\nabla \mathbf{v}^f + \nabla \mathbf{v}^{f^T} \right) : \nabla \mathbf{v}^f + \rho^f \frac{C_2 \varepsilon^2}{k} = 0 \tag{5}$$

$$L = \frac{k^{3/2}}{\varepsilon}. \tag{6}$$

Typical constants of k–ε model of Launder and Spalding [20] are $C_\mu = 0.09$, $C_1 = 1.44$, $C_2 = 1.92$, $\sigma_k = 1.0$ and $\sigma_\varepsilon = 1.0$.

Eqs. (2)–(6) are solved implicitly using a standard isoparametric finite element discretization for $\mathbf{v}^f$, k and ε. The incompressibility constrain – Eq. (1) – is imposed by penalization [21]. A streamline Upwind Petrov–Galerkin technique [22] is used for stabilization.

The iterative scheme required to solve the equations use the k-predictor / (ε) corrector algorithm described in [23,24] together with wall functions for boundary conditions [25].

3. Particle transport model

As mentioned in the introduction, the coupling between the two phases depends on the concentration of particles. We assume that concentration of particles is low enough in order to consider the one-way coupling model as valid. This constitutes an important simplification since the fluid and particle dynamics are decoupled and the particle motion is calculated once the fluid flow has been obtained.

The motion of a rigid particle in a viscous flow may be described by an ordinary differential equation derived by Oseen based upon the works of Boussinesq and Basset. When the effect of the previous history of the particle (Basset force term) is neglected, the BBO (Basset–Boussinesq–Oseen) equation may be expressed [26,27].

$$\rho^p \frac{d\mathbf{v}^p}{dt} = \rho^f \frac{d\mathbf{v}^f}{dt} + \left(\rho^p - \rho^f \right) \mathbf{g} + \frac{3}{4} \frac{\rho^f}{D^p} C_D \left(\mathbf{v}^f - \mathbf{v}^p \right) |\mathbf{v}^f - \mathbf{v}^p| - \frac{\rho^f}{2} \frac{d\left(\mathbf{v}^p - \mathbf{v}^f \right)}{dt} \tag{7}$$

Eq. (7) represents the Newton law for a spherical particle of density ρ^p, diameter D^p and velocity $\mathbf{v}^p$. Terms on the right hand side are volume forces acting on the particle. Four different forces are taken into account in Eq. (7):

- the inertial force (first term on the right hand side) which depends on local fluid velocity $\mathbf{v}^f$ and fluid density ρ^f,
- the buoyancy force (second term on the right hand side) due to the action of gravity acceleration,
- the drag force (third term on the right hand side), characterized by the coefficient C_D – to be discussed below – and