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Transient freezing of liquids in forced flow inside convectively cooled tubes ☆

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Abstract

The aim of this investigation is to study the characteristics of solidification of a liquid flowing through a convectively cooled pipe under different flow situations. A mathematical model is developed by establishing an energy balance. The amount of heat transferred from the liquid layer to the freeze-front in a tube section is directly proportional to the convective heat transfer coefficient and the difference between the average temperature of the fluid at the section and the freezing point. The limiting conditions for the commencement of the solidification process are stated. The formulated set of conjugated heat transfer equations is analysed for both laminar and turbulent flows. The governing equations are solved numerically for a specific range of parameters and their characteristics of the process discussed.

Keywords: Pipe fluid flow; Freezing; Laminar and turbulent cases; Cooling parameter; Stefan and Biot numbers

1. Introduction

The solidification of liquids flowing through pipes and nozzles under laminar and turbulent flow conditions and also of liquids surrounding a pipe is of practical interest. This includes the casting of metals, freezing in water mains and also this technique is used in onshore/offshore pipe lines to provide a flow blockage thereby isolating the defective section for servicing. The study of axial growth of solid layer inside pipes is essential to predict the pressure built-up in the flow circuit. A variety of studies dealing with analytical, numerical and experimental aspects of the above process have appeared for different flow situations and boundary conditions.

Thomasson et al. [1] have reported some of the earlier works and presented analytical and experimental results for the steady state annular freezing for turbulent flows. A few of the pertinent earlier works referred here are Zerkle and Sunderland [2], Shibani and Ozisik [3], DesRuisseaux and Zerkle [4], and Epstein and Cheung [5]. Seeniraj and Velraj [6] reported the characteristics of solidification of a stationary fluid-liquid phase change material-inside a finned tube under convective boundary condition and also of liquid outside a tube [7]. The purpose of the present work is to provide numerical solutions for transient freezing of warm liquids flowing through a convectively cooled pipe under laminar and turbulent conditions. The present formulation brings in a new cooling parameter related to Biot number and this provides the condition for onset of freezing of a flowing warm liquid. It is shown that, in the limiting case the present results agree with those of Epstein and Cheung [5] reported for steady state and constant temperature boundary. During the analysis of this problem with the natural mixed boundary conditions, limits for some important parameters, which govern the commencement of solidification, are also reported. The influence of various parameters on the axial growth of solid layer at different stages of freezing is discussed here.

2. Problem formulation and analysis

The present investigation considers the transient solidification of a warm-liquid flowing through a convectively cooled pipe (with the assumptions stated in Epstein and Cheung [5]). Warm liquid from a reservoir enters the pipe at $z=0$ with a uniform temperature, T_{in} greater than its freezing temperature, T_{f}. A coolant at temperature, $T_\infty < T_{\text{f}}$, with a constant convection

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Nomenclature

A	non-dimensional cooling parameter
$A_{i,\,o}$	inner, outer surface area of the pipe
A_m	log mean area of A_i and A_o
Bi	Biot number
C_p	specific heat
Ei	exponential integral
h	convective heat transfer coefficient
h_{sf}	latent heat of solidification
k	thermal conductivity
m	mass flow rate
Nu	Nusselt number
Pe	Peclet number
Pr	Prandtl number
r	radial co-ordinate
R	non-dimensional radial co-ordinate
Re	Reynolds number
Ste	Stefan number
T	temperature
u	mean velocity of fluid
U	overall heat transfer coefficient
z	axial co-ordinate
Z	non-dimensional axial coordinate

Greek Symbols

α	thermal diffusivity
δ	thickness
θ	non-dimensional temperature
ν	kinematic viscosity
ρ	density
τ	non-dimensional time
ξ	scaled axial co-ordinate

Subscripts

b	bulk mean value for fluid
f	freezing point/location of freeze-front
i	inner surface of the pipe
in	pipe inlet
l	liquid phase
o	outer surface of the pipe
s	solid phase
sl	ratio of solid property to liquid property
w	pipe wall
z	axial location
∞	outside coolant

coefficient h_∞, convectively cools the exterior of the pipe. There exits a precooling stage before solidification begins. Fig. 1 shows the schematic of the problem at time, $t>0$.

The system of governing equations that follow, are stated in non-dimensional form by introducing the following variables and parameters:

Bi $= U_i r_i / k_s$
Nu $= 2 h_i r_f / k_l$
Pr $= \gamma_1 / \alpha_1$
R $= r/r_i$
Re $= 2 u_z r_i / \gamma_1$
$\mathrm{Ste_s} = (T_f - T_\infty) C_{ps} / h_{ls}$
$\mathrm{Ste_l} = (T_{in} - T_f) C_{pl} / h_{ls}$
$\mathrm{Ste_{sl}} = \mathrm{Ste_s}/\mathrm{Ste_l}$
Z $= z/r_i$
$\theta_b = (T_b - T_f)/(T_{in} - T_f)$
$\theta_s = (T_f - T_s)/(T_f - T_\infty)$
$\tau = \alpha_s t / r_i^2$

An instantaneous energy balance established over an elemental control volume shows that the enthalpy drop of the flowing liquid is equal to the energy convected from the liquid to the growing solid layer.

$$\mathrm{Pe}\left(\frac{\partial \theta_b}{\partial Z}\right) + 2\mathrm{Nu}\theta_b = 0 \tag{1}$$

θ_b represents the fluid non-dimensional bulk mean temperature at $Z>0$. Eq. (1) is based on a quasi-steady state assumption that the fluid bulk mean temperature reaches a steady state distribution instantaneously as the freeze-front radius changes. This is a reasonable assumption for fluids having a high Peclet number. A constant displacement pump maintains a constant liquid flow rate. The steady mass flow continuity which provides a relation between the local mean velocity u_z ($Z>0$) and the inlet velocity u_{in} ($Z=0$) as $u_z/u_{in} = (r_i/r_f)^2$, is used in the axial variation of bulk mean temperature.

The localized asymptotic Nusselt number, Nu(R_f) correlation in [8] employed in the internal solidification for turbulent flow is

$$\mathrm{Nu} = 0.0155\mathrm{Re}^{0.83}\mathrm{Pr}^{0.5}\left(\frac{1}{R_f^{0.83}}\right) \tag{2}$$

and for laminar flow within a tube at constant temperature is constant. At $\tau>0$, the surface of the cylindrical solid shell through which the liquid flows is at constant temperature, T_f. The constant value for the Nusselt number makes the axial variation of the bulk mean temperature independent of time and the axial solid layer growth would be hardly noticeable.

At the interface the sum of the heat released due to solidification and the heat flux from the warm liquid to the frozen layer is equal to the heat flux conducted through the solid layer, which is finally absorbed by the coolant surrounding the pipe. This interface radial energy balance, different from the earlier works, considers the localized convective heat flux through Nu (R_f) and $\theta_b = \theta_b$ (Z, τ)

$$\left(\frac{\partial R_f}{\partial \tau}\right) = -Ste_s\left(\frac{\partial \theta_s}{\partial R}\right)_{R=R_f} + \left(\frac{\phi\theta_b}{R_f}\right) \tag{3}$$

where ϕ is

$$\phi(R_f) = \left(\frac{C_p}{2k}\right)_{sl}\mathrm{Ste_l}\mathrm{Nu}(R_f) \tag{3a}$$