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Unsteady MHD free convection of a micropolar fluid between two parallel porous vertical walls with convection from the ambient ☆

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ABSTRACT

The present work is concerned with the unsteady free convection flow of an incompressible electrically conducting micropolar fluid, bounded by two parallel infinite porous vertical plates submitted to an external magnetic field and the thermal boundary condition of forced convection. The governing equations are solved using a numerical technique based on the electrical analogy, where only previous spatial discretization is necessary to obtain a stable and convergent solution with very low computational times. To solve the system of algebraic equations with time as continuous function, an electric circuit simulator is used. This method permits the direct visualization of the local and/or integrated transport variables (temperatures and velocities) at any point or section of the medium. Numerical results for temperature, velocity and microrotation are illustrated graphically.

1. Introduction

The dynamics of micropolar fluids has attracted considerable attention during the last few decades because traditional Newtonian fluids cannot precisely describe the characteristics of fluid flow with suspended particles. Erigen [1] developed the theory that the local effects arising from the microstructure and the intrinsic motion of the fluid elements should be taken into account. The theory is expected to provide a mathematical model for the Non-Newtonian fluid behaviour observed in certain man-made liquids such as polymers, lubricants, fluids with additives, paints, animal blood and colloidal and suspension solutions, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. Later, Erigen [2] extended the theory of thermo-micropolar fluids and derived the constitutive laws for fluids with microstructures.

Micropolar fluids have recently received considerable attention due to their potential application in many industrial processes; for example, in continuous casting glass-fiber production, paper production, metal extrusion, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion and metal spinning. Balaram and Sastry [3] solved the problem of a fully developed free convection flow in a micropolar flow. Agarwal and Dhanapal [4] obtained a numerical solution to study the fully developed free convection micropolar fluid flow between two parallel with constant suction (or injection). Srinivasacharya et al. [5] studied the effects of microrotation and frequency parameters on an unsteady flow of micropolar fluid between two parallel porous plates with a periodic suction. El-Hakiem [6] obtained a similarity solution for the flow of a micropolar fluid along an isothermal vertical plate with an exponentially decaying heat generation term and thermal dispersion.

When magneto-hydrodynamic effects are added to the microrotation, an interesting new problem arises due to several engineering applications such as in MHD electrical power generation, designing cooling system for nuclear reactors, etc., where microrotation provides an important parameter for deciding the rate of heat flow. Gorla et al. [7] developed a numerical scheme to solve the steady free convection from a vertical isothermal plate in a strong cross magnetic field immersed in a micropolar fluid. El-Hakiem et al. [8] analysed the effect of viscous and Joule heating on the flow of an electrically conducting and micropolar fluid past a plate whose temperature varies linearly with the distance from the leading edge in the presence of a uniform transverse magnetic field. Helmy et al. [9] studied the unsteady flow MHD of a conducting micropolar fluid, through a porous medium, over an infinite plate that is set in motion in its own plane by an impulse. Bhargara et al. [10] obtained a numerical solution of a free convection MHD micropolar fluid flow between two parallel porous vertical plates by means of the quasi-linearization method. By means of a similarity transformation and using the numerical scheme of Chebyshev finite difference, Eldabe and Mahmoud [11] solved the problem of a flow past a stretching surface with both heat and mass transfer, Ohmic heating and viscous dissipation.

The main objective of this study was to analyse the unsteady free convection from two parallel porous vertical plates that exchange heat with an external fluid. Between these plates an electrically conducting micropolar fluid is placed in the presence of a strong cross magnetic field.

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Nomenclature

B	dimensionless micropolar parameter,ce:section>j/L^2
Bi	Biot number
C	dimensionless micropolar parameter, $L^2/(1+\Lambda)$
C	capacitor, F
c_p	specific heat, J kg^{-1} K^{-1}
h	heat transfer coefficient, Wm^{-2} K^{-1}
Ha	Hartmann number, $H_0\ (\sigma/\mu)^{1/2}$
H_0	magnetic field intensity, We m^{-2}
j	micro-inertia density, m^2
k	thermal conductivity, W m^{-1} K^{-1}
K	rotational viscosity coefficient, kg s^{-1} m^{-1}
n	angular velocity, rad s^{-1}
N	dimensionless angular velocity, $n\,\rho\,g\,\beta\,L^3\,\upsilon\,n/k$
g	acceleration due to gravity, m s^{-2}
G	control-voltage current-source
J	electrical intensity, A
L	thickness of the channel, m
Pr	Prandtl number, υ/α
Q	heat generation or absorption, W m^{-3} K^{-1}
R	resistor, Ω
Re	Reynolds number, $v_0\,\rho\,L/\mu$
t	time, s
T	temperature, K
T_0	temperature in hydrostatic state, K
u	velocity in x-direction, m s^{-1}
U	dimensionless velocity, $u\,\rho\,g\,\beta\,L^2/k$
v	velocity in y-direction, m s^{-1}
v_0	suction/injection velocity, m s^{-1}
x	axial co-ordinate, m
y	perpendicular co-ordinate, m
Y	dimensionless co-ordinate, y/L

Greek symbols

Φ	voltage, V
α	thermal diffusivity, m^2 s^{-1}
β	volumetric coefficient of thermal expansion, K^{-1}
γ	microrotational coupling coefficient, N s
ΔY	thickness of the cell
ε	non-dimensional heating parameter, $(T_2-T_0)/(T_1-T_0)$
ζ	dimensionless group, $Pr\,Gr\,\beta\,g\,L/c_p$
θ	dimensionless temperature, $(T-T_0)\,\rho^2\,g^2\,\beta^2\,L^4/k\,\mu$
λ	dimensionless micropolar parameter, $\gamma/\mu\,L^2$
γ_1	dimensionless micropolar parameter, $\gamma_0\,L/\rho\,c_p$
Λ	dimensionless micropolar parameter, K/μ
μ	dynamic viscosity, N s m^{-2}
ρ	density, kg m^{-3}
σ	electrical conductivity, Ω^{-1} m^{-1}
τ	dimensionless time, $t\,\upsilon/L^2$
υ	kinematic viscosity, m^2 s^{-1}

Subscripts

$j,\ j-\Delta,\ j+\Delta$	associated to the centre, left and right position on the cell
w	condition at the wall

The analysis is based on numerical solution of full governing equations using the Network Simulation Method (NSM). Numerical results are given graphically for the velocity, microrotation and temperature unsteady and steady-state profiles when Λ (material parameter), Re (cross flow Reynolds number), γ_1 (heat generated parameter), Ha (Hartmann number), Bi_1 and Bi_2 (Biots numbers), Pr (Prandtl number) are modified. The effect of Joule heating is also studied.

NSM initially requires spatial discretization to be applied to the transient boundary-layer equations, thus providing a set of ordinary differential equations, one for each control volume. With the application of the electrical–thermal-motion analogy an elemental network model (elemental cell or elemental control volume) is developed, which, extended to all the medium, together with the boundary conditions, is solved by means of a program commonly used to simulate electrical circuits, Pspice [12]. The main advantage of the method is that time derivatives are not replaced by finite differences (similar to the method of lines [13]), but only require finite-difference schemes for the spatial variable. In this way, the time remains as a continuous variable, which results in greater accuracy and no time interval needs to be established by the programmer. Besides, the method does not require convergence criteria to solve the finite difference equations resulting from the discretization of the partial difference equations of the mathematical model, since the powerful software Pspice does the work.

2. Mathematical model

Consider an unsteady, laminar, fully developed free convection flow of an incompressible micropolar fluid flowing between two infinite parallel porous vertical walls submitted to a strong magnetic field H_0 in the direction normal to the plate. Both the walls will be assumed to have a negligible thickness and to exchange heat with an external fluid by convection. Outside the plate, there is a quiescent ambient fluid at a constant temperature T_∞ with u and v denoting, respectively, the velocity components in the x and y direction, where x is vertically upwards and y is the coordinate perpendicular to x. There is a component of microrotation in the direction normal to x and y, (0,0,n). All fluid properties are considered to be constant except for the density variation which induces the buoyancy force. The transient, two-dimensional flow can be shown to be governed by the following boundary layer equations,

Continuity equation:

$$\partial v/\partial y = 0 \tag{1}$$

The integration of Eq. (1) gives $v = v_0$ (constant)

Momentum equation:

$$\partial u/\partial t + v_0 \partial u/\partial y = (\upsilon + K/\rho)\partial^2 u/\partial y^2 + \beta g(T-T_0) + K/\rho\,\partial n/\partial y - u\,\sigma H_0^2/\rho \tag{2}$$

Energy of angular moment:

$$\rho j\,\partial n/\partial t + v_0 \rho j\,\partial n/\partial y = \gamma\,\partial^2 n/\partial y^2 - K(2n + \partial u/\partial y) \tag{3}$$

Energy equation:

$$\rho c_p \partial T/\partial t + \rho c_p v_0\,\partial T/\partial y = k\,\partial^2 T/\partial y^2 + (\mu + K)(\partial u/\partial y)^2 + \gamma(\partial n/\partial y)^2 + 2K(n^2 + n\partial u/\partial y) + Q + u^2\sigma H_0^2 \tag{4}$$

with the following initial and boundary conditions:

$$\text{For } t \le 0;\ u = 0, n = 0, T = T_0 \tag{5a}$$

$$\text{For } t > 0;\ u = 0, v = v_0, n = 0, -k_w \partial T/\partial y_{(y=0)} = h_1\left[T_1 - T_{(y=0)}\right] \text{ at } y = 0 \tag{5b}$$

$$u = 0, v = v_0, n = 0, -k_w \partial T/\partial y_{(y=L)} = h_2\left[T_{(y=L)} - T_2\right] \text{ at } y = L \tag{5c}$$

where ρ is the density, c_p the specific heat, h the convective coefficient, k the thermal conductivity of the fluid, υ the kinematic viscosity, μ the dynamic viscosity, K the gyroviscosity, γ the material constant, j the microinertia, σ the electrical conductivity of the fluid and T_0 is the temperature in hydrostatic state. $Q = \gamma_0\ v_0\ (T - T_0)$ is the volumetric