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A simple mathematical model for determining the equivalent permeability of fractured porous media☆

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ABSTRACT

A simple mathematical model has been proposed so as to determine the equivalent permeability of fractured porous media. The model consists of square blocks placed in an array with vertical and horizontal fractures between the blocks. An analytical expression valid for all macroscopic flow directions has been derived for the equivalent permeability of the fractured porous media, assuming a horizontal flow through the blocks placed in a porous medium. The analytical expression agrees well with the existing equations and also with the microscopic numerical results carried out using a unit structure with periodic boundary conditions. The foregoing two-dimensional model has been extended to a three dimensional case in which the cubic rocks are arranged in a cubic array. The resulting three-dimensional analytical expression for the equivalent permeability is found to agree very well with both existing formula and microscopic numerical simulation.

1. Introduction

A several conceptual models have been proposed for describing fluid flows in fractured porous media. Bear and Berkowitz [1] classified these models as the *near field*, where flow occurs in a fractured porous medium and each fracture is described in detail; the *far field*, where flow occurs in two overlapping continua with mass exchanged through coupling parameters; and the *very far field*, where fracture flow occurs, on average, in an equivalent porous medium. The discrete fracture models such as used by Travis [2], Shimo and Long [3] and Sudicky and McLaren [4] belong to the first class, namely, the near field model. As the number of fractures increases, the computational burden of the near field models increases enormously. The second class, the far field model, corresponds to the dual-continuum approaches, which were introduced by Barenblatt et al. [5] and later extended by Warren and Root [6]. In this model, the porous medium and the fractures are envisioned as two separate but overlapping continua. The mass transfer between the fracture and medium is prescribed by introducing a coupling parameter. The very far field model, namely, equivalent continuum model, is the simplest but quite effective model, when dealing with a sufficiently large field of flows in fractured porous medium. In this model, we introduce an equivalent permeability concept and treat a volume of interest, as a single equivalent continuum. An excellent review on these conceptual models may be found in Diodato [7]. As pointed out by Liu et al. [8,9], the validity of the far field model solely depends on the way to determine the equivalent permeability.

In this paper, we shall focus on the equivalent continuum model and propose a rational mathematical model for determining the equivalent permeability of the fractured porous medium. We propose a simple mathematical model to determine the equivalent permeability of fractured porous media, in which the fractured porous medium is modeled as an array of square blocks placed in a fluid saturated porous medium. The absolute permeability of the blocks (i.e. rocks) is assumed much smaller than that of the fractures. Upon assuming a horizontal flow through fractures, an analytical expression is derived for determining the equivalent permeability of the fractured porous media. A series of numerical calculations are conducted using a single structural unit with periodic boundary conditions. Then, the equivalent permeability is evaluated from the microscopic velocity and pressure fields so as to examine the validity of the analytical expression. This two-dimensional model will be extended to a three dimensional case in which the cubic rocks are arranged in a cubic array. The resulting analytical expression for the equivalent permeability is found to agree very well with both existing formula and microscopic numerical simulation.

2. Mathematical model

We consider a dual structured porous medium as shown in Fig. 1. The square blocks represent the rocks of size D, while the passages between them indicate the fractures of aperture d filled with a porous medium of the absolute permeability K_f and the porosity ε_f.

The present model accounts for both horizontal and vertical fractures, and thus differs from those models having only horizontal fractures. However, the effective horizontal permeability obtained assuming a

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Nomenclature

d	fracture aperture
D	size of blocks
H	size of unit structure
$\vec{i}, \vec{j}$	unit vectors
K	permeability
p	intrinsic pressure
$Re = u_B H/\nu$	Reynolds number based on the bulk velocity and size of unit structure
s	coordinate measured along the macroscopic flow direction
u, v	velocity components
u_B	bulk velocity
u_D	Darcian velocity
$\vec{u}$	local velocity vector
x, y, z	Cartesian coordinates
$\alpha = (d/2)\sqrt{\varepsilon_f/K_f}$	parameter associated with the fracture porosity and permeability
ε	porosity
ν	kinematic viscosity of fluid
θ	macroscopic flow angle
ρ	density of fluid

Subscripts

e	effective
f	fracture
m	rock matrix

horizontal flow through the present block model may not be very far from that obtained with the model having only horizontal fractures. Since the permeability of the fractures is much higher than that of the rock, most fluids choose to pass through the fractures. The advantage of the present model having both horizontal and vertical fractures will be clear as we generalize the results for the effective directional permeability.

3. Numerical model

The governing equations of continuity and momentum as introduced by Vafai and Tien [10] are written for the flow through fractures filled with a porous medium of the absolute permeability K_f and the porosity ε_f as follows:

$$\nabla \cdot \vec{u} = 0 \quad (1)$$

$$\frac{1}{\varepsilon_f^2}\left(\vec{u}\cdot\nabla\right)\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\nu}{\varepsilon_f}\nabla^2\vec{u} - \frac{\nu}{K_f}\vec{u} \quad (2)$$

where $\vec{u}$ is the local volume average velocity vector whereas p is the intrinsic average pressure within the fractures. When $\varepsilon_f \to 1$ and $K_f \to \infty$, Eq. (2) naturally reduces to the Navier–Stokes equation. The boundary and compatibility conditions are given by

On the solid walls:

$$\vec{u} = \vec{0} \quad (3)$$

On the periodic boundaries:

$$\vec{u}\,|_{x=-H/2} = \vec{u}\,|_{x=H/2} \quad (4a)$$

$$\vec{u}\,|_{y=-H/2} = \vec{u}\,|_{y=H/2}. \quad (4b)$$

The previous study [8] reveals that the periodic boundary conditions are found sufficient for evaluating the pressure drops. The governing Eqs. (1) and (2) are readily discretized by integrating them over a grid volume. SIMPLE algorithm for the pressure–velocity coupling, as proposed by Patankar and Spalding [11] can be adopted to correct the pressure and velocity fields. Calculation starts with solving the two momentum equations, and subsequently, the estimated velocity field is corrected by solving the pressure correction equation reformulated from the discretized continuity and momentum equations, such that the velocity field fulfills the continuity principle. This iteration sequence must be repeated until convergence is achieved. Convergence can be measured in terms of the maximum change in each variable during an iteration. The maximum change allowed for the convergence check may be set to an arbitrarily small value (such as 10^{-5}), as the variables are normalized by appropriate references. A fully implicit scheme may be adopted with the hybrid differencing scheme for the advection terms. Calculations are carried out using highly nonuniform grid arrangements such as 100×100. Further details on this numerical procedure can be found in Patankar [12] and Nakayama [13].

4. Effective horizontal permeability

Assuming a channel flow through horizontal fractures filled with the porous medium of absolute permeability K_f and absolute porosity ε_f, we may readily reduce Eqs. (1) and (2) to

$$-\frac{1}{\rho}\frac{dp}{dx} + \frac{\nu}{\varepsilon_f}\frac{d^2u}{dy^2} - \frac{\nu}{K_f}u = 0 \quad (5)$$

Thus, we may estimate the pressure gradient across the fractured porous medium, using the expression obtained by Nakayama et al. [14] on the basis of Kaviany's Brinkman–Darcy solution [15]:

$$-\frac{dp}{dx} = \frac{4\mu}{d^2}\left(\frac{u_B}{\varepsilon_f}\right)\frac{\alpha^3\cosh\alpha}{\alpha\cosh\alpha - \sinh\alpha} \quad (6)$$

where u_B is the bulk velocity and

$$\alpha = \frac{d}{2}\sqrt{\frac{\varepsilon_f}{K_f}} \quad (7)$$

is the dimensionless number related to the fracture aperture d and square root of absolute fracture permeability K_f. We note that the Darcian velocity is related to the bulk velocity as

$$u_D = \frac{d}{H}u_B = \left(1 - \frac{D}{H}\right)u_B = \left(1 - (1-\varepsilon_m)^{1/2}\right)u_B \quad (8)$$

and

$$\frac{D}{d} = \frac{(1-\varepsilon_m)^{1/2}}{1-(1-\varepsilon_m)^{1/2}} \quad (9)$$

where $\varepsilon_m = (H^2 - D^2)/H^2$ is the volume fraction occupied by the fractures (i.e. fracture porosity). Hence, the analytical expression for the effective horizontal permeability K_e may be given by

$$K_e = \frac{\mu u_D}{\left(-\frac{dp}{dx}\right)} = \frac{\left(1-(1-\varepsilon_m)^{1/2}\right)^3\varepsilon_f}{4(1-\varepsilon_m)}\left(\frac{\alpha\cosh\alpha - \sinh\alpha}{\alpha^3\cosh\alpha}\right)D^2. \quad (10)$$

For the limiting case of $\alpha \to 0$ and $\varepsilon_f \to 1$ (i.e. clear fluid flows through fractures without porous media), Eq. (10) reduces to:

$$K_e|_{\alpha\to 0} = \frac{\left(1-(1-\varepsilon_m)^{1/2}\right)^3}{12(1-\varepsilon_m)}D^2 \qquad (0.08 \le \varepsilon_m \le 0.84) \quad (11)$$