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Stability of thermal transpiration flows in rectangular enclosures – Axisymmetric disturbances ☆

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Abstract

Occurrence of instabilities for thermal transpiration flow of rarefied gases has been discussed in two-dimensions. Only axisymmetric disturbances have been considered due to symmetry of the basic flow. Effect of four second-order slip models (Cercignani, Deissler, Schamberg, and Beskok) and a first-order slip model (Maxwell) on the limits of the proposed instability have been examined. We have found that Beskok model is always stable to axisymmetric disturbances and Maxwell model is more stable compared with the other second order models. Variation of critical Reynolds number for different rarefaction levels (Knudsen numbers) has also been studied. Increase in Knudsen number leads to monotonic decrease in critical *Re* numbers.

Keywords: Linear stability; Thermal transpiration; Rarefied gases; Axisymmetric disturbances

1. Introduction

Nowadays, there is an increasing attention to convection of rarefied gases due to rapid emergence of MEMS devices. Heat transfer to rarefied gases leads to variety of motions [1]. Buoyancy driven and thermal transpiration flows are only two examples of these flows. Instabilities of the former flow is primarily induced by buoyancy forces resulting from a temperature gradient between bottom and top boundaries of a closed rectangular geometry. This phenomenon has been extensively studied by many researchers under the heading of Rayleigh–Bénard (RB) instability [2–4]. Stefanov et al. [5,6] have investigated long-time behaviour of the RB convection of rarefied gases for varying Knudsen ($1\times10^{-3}<Kn<4\times10^{-2}$) and Froude ($1\times10^{-3}<Fr<1.5\times10^{3}$) numbers. Both Direct Simulation Monte Carlo (DSMC) and direct simulations of Navier Stokes (NS) equations have been performed to identify zone of instabilities within the Kn–Fr parameters space. Golshtein and Elperin [7] have pointed out that Boussinesq approximation becomes invalid if the temperature difference (T_h-T_c) between the bottom and top surfaces is high enough, and limits of RB instability cannot be predicted only by Rayleigh (Ra) number. Recently, a linear stability analysis of RB convection of compressible gases in the slip flow regime has been studied by Manela and Frankel [8]. They have reported a close constitution between the linear stability and DSMC analyses of reference [5] on the threshold of the instability.

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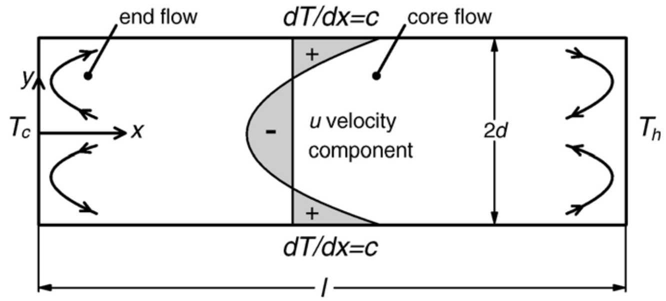

Fig. 1. Thermal creep flow in a rectangular enclosure. The flow takes place in positive *x* direction, and it turns to reverse direction on the vertical walls to satisfy conservation of mass. The flow domain can be divided into two main regions: core flow and end flow regions. In the former region, streamlines of the flow are almost parallel to the horizontal walls. On the other hand, components of the velocity vector show large gradients in the vicinity of the ends.

Thermal transpiration is another mechanism inducing motion of rarefied gases. Non-isothermal temperature of a surface bounding a rarefied gas produces thermal transpiration flow. Thermally driven motion of rarefied gases has been first recognized by Reynolds and then a preliminary theoretical effort investigating the reasons of thermal transpiration has been devoted by Maxwell [9]. A flow generated by thermal creep along horizontal walls of an enclosure is shown in Fig. 1. Papadopoulos and Rosner [10] have pointed out significance of this motion in the given geometry even for non-negligible gravity conditions. Orhan [11] has also studied this motion by continuum models (NS and augmented Burnett) numerically and observed that slip velocity on the upper wall increases nonlinearly up to a limit of *Kn* number and then becomes nearly constant with further increase in *Kn* number.

Another kind of surface driven flow, but not in the class of rarefied gas flows, observed in shallow liquids is *thermocapillary convection* (instability). Traction forces resulting from temperature dependent surface tension create such a fluid instability as first realized by Pearson [12]. An overview and comprehensive details of the thermocapillary convection are summarized by Davis [13]. Notice that, there is a close similarity between the thermal transpiration and thermocapillary flows. Both flows are induced by non-isothermal variation of surface temperatures.

Due to this similarity, current study concentrates on occurrence of thermal transpiration instabilities of rarefied gases in rectangular enclosures. For this purpose, we introduce physical and mathematical aspects of the model used in Section 2. Results of the analyses and mechanism of the possible instability will be discussed in Section 3.

2. Physical and mathematical modeling

Let us consider circulation of rarefied gas in an enclosure with an aspect ratio as illustrated in Fig. 1. While the left side of the enclosure is maintained at the temperature of T_c, the right side is kept at the temperature of T_h ($T_h > T_c$). Temperature gradients on the top and bottom walls are constant such that $\frac{dT}{dx} = c =$ constant, $c>0$. Due to linearly varying horizontal wall temperatures, a motion of rarefied media starts to evolve from the cold to hot region. It is obvious that axisymmetry of the basic flow enables us to analyse stability of the proposed flow to axisymmetric disturbances.

The rarefied gas has been assumed as incompressible. Such an assumption holds even for moderate temperature difference between the top and bottom walls if $Ma \leq 0.3$ as in the case of Couette flow discussed by Beskok et al. [14]. Unsteady and incompressible NS model in three-dimensions is written for continuity;

$$\text{div}\,\mathbf{u} = 0, \tag{1}$$

momentum;

$$Re\left[\frac{\partial \mathbf{u}}{\partial \tau} + (\mathbf{u}\cdot\text{grad})\mathbf{u}\right] = -\text{grad}\,p + \nabla^2\mathbf{u} \tag{2}$$

and energy equation;

$$RePr\left[\frac{\partial \theta}{\partial \tau} + (\mathbf{u}\cdot\text{grad})\theta\right] = \nabla^2\theta. \tag{3}$$